# Appendix C
# Stability Analysis Procedures - Theory and Limitations

## C-1. Fundamentals of Slope Stability Analysis

*a. Conventional approach.* Conventional slope stability analyses investigate the equilibrium of a mass of soil bounded below by an assumed potential slip surface and above by the surface of the slope. Forces and moments tending to cause instability of the mass are compared to those tending to resist instability. Most procedures assume a two-dimensional (2-D) cross section and plane strain conditions for analysis. Successive assumptions are made regarding the potential slip surface until the most critical surface (lowest factor of safety) is found. Figure C-1 shows a potential slide mass defined by a candidate slip surface. If the shear resistance of the soil along the slip surface exceeds that necessary to provide equilibrium, the mass is stable. If the shear resistance is insufficient, the mass is unstable. The stability or instability of the mass depends on its weight, the external forces acting on it (such as surcharges or accelerations caused by dynamic loads), the shear strengths and porewater pressures along the slip surface, and the strength of any internal reinforcement crossing potential slip surfaces.

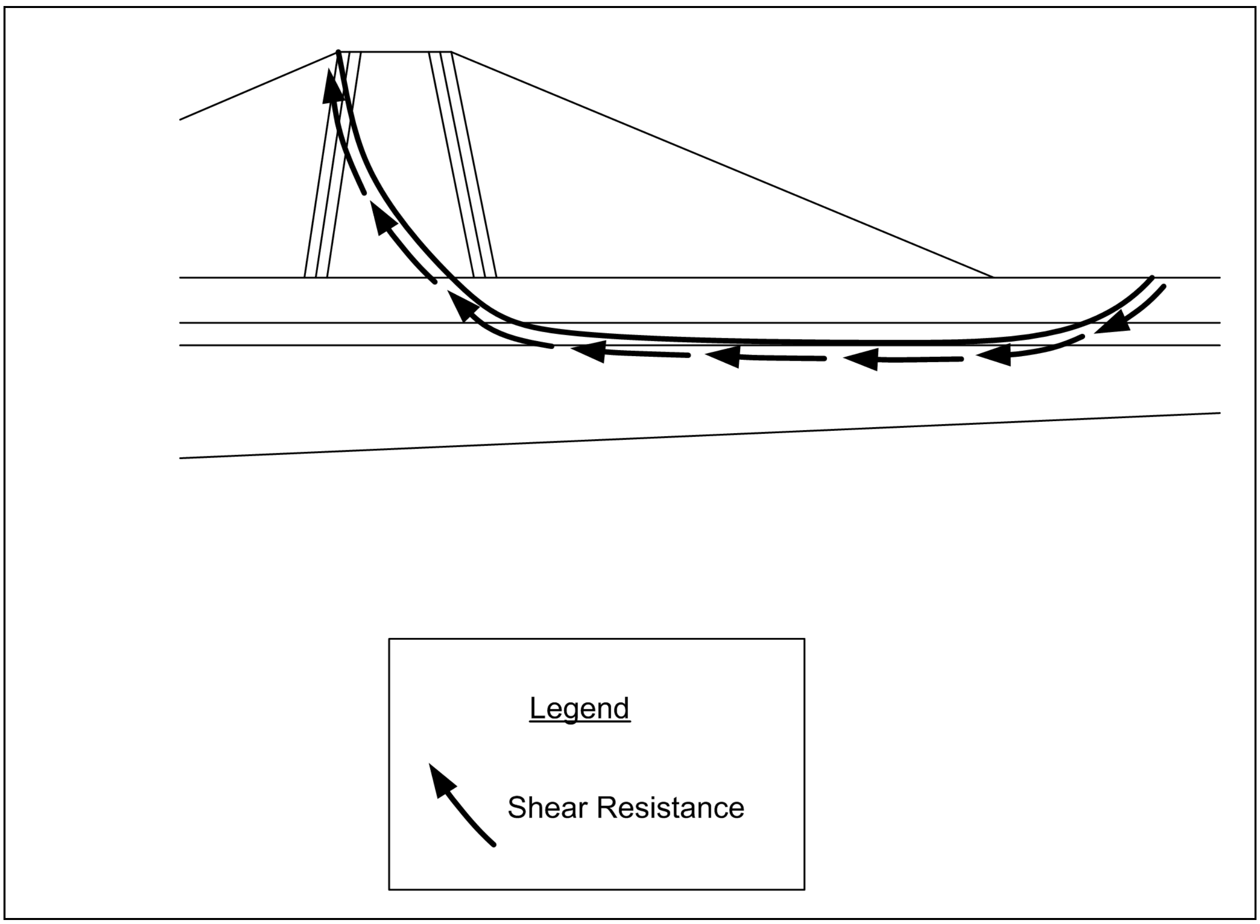



**Figure C-1. Slope and potential slip surface**

*b. The factor of safety.* Conventional analysis procedures characterize the stability of a slope by calculating a factor of safety. The factor of safety is defined with respect to the shear strength of the soil as the ratio of the available shear strength (s) to the shear strength required for equilibrium ($\tau$), that is:

$$F = \frac{\text{Available shear strength}}{\text{Equilibrium shear stress}} = \frac{s}{\tau} \tag{C-1}$$

(1) Shear strength is discussed further in Appendix D. If the shear strength is defined in terms of effective stresses, the factor of safety is expressed as:

$$F = \frac{c' + (\sigma - u)\tan\phi'}{\tau} \tag{C-2}$$

where

$c'$ and $\phi'$ = Mohr-Coulomb cohesion and friction angle, respectively, expressed in terms of effective stresses

$\sigma$ = total normal stress on the failure plane

u = pore water pressure; $(\sigma - u)$ is the effective normal stress on the failure plane

If the failure envelope is curved, the factor safety can be expressed as:

$$F = \frac{s\ (\sigma')}{\tau} \tag{C-3}$$

where s $(\sigma')$ represents the shear strength determined from the effective stress failure envelope for the particular effective normal stress, $\sigma'$.

Equation C-2 can also still be used with a curved failure envelope by letting $c'$ and $\phi'$ represent the intercept and slope of an equivalent linear Mohr-Coulomb envelope that is tangent to the curved failure envelope at the appropriate value of normal stress, $\sigma'$.

(2) For total stresses, the factor of safety is expressed using the shear strength parameters in terms of total stresses, i.e.:

$$F = \frac{c + \sigma\tan\phi}{\tau} \tag{C-4}$$

where c and $\phi$ are the Mohr-Coulomb cohesion and friction angle, respectively, expressed in terms of total stresses. Curved failure envelopes are handled for total stresses in much the same way they are handled for effective stresses: The strength is determined from the curved failure envelope using the particular value of total normal stress, $\sigma$. In the remaining sections of this Appendix, the effective stress form of the equation for the factor of safety (Equation C-2) will be used. Any of the equations presented in terms of effective stress can be converted to their equivalent total stress form by using c and $\phi$, rather than $c'$ and $\phi'$, and by setting pore water pressure, u, equal to zero.

*c. Limit equilibrium methods – General assumptions.* All of the methods presented in this manual for computing slope stability are termed "limit equilibrium" methods. In these methods, the factor of safety is calculated using one or more of the equations of static equilibrium applied to the soil mass bounded by an assumed, potential slip surface and the surface of the slope. In some methods, such as the Infinite Slope method, the shear and normal stresses ($\sigma$ and $\tau$) can be calculated directly from the equations of static equilibrium and then used with Equation C-2 or C-4 to compute the factor of safety. In most other cases, including the Simplified Bishop, the Corps of Engineers' Modified Swedish Method, and Spencer's Method, a more complex procedure is required to calculate the factor of safety. First, the shear stress along the shear surface is related to the shear strength and the factor of safety using Equation C-2 or C-4. In the case of effective stresses, the shear stress according to Equation C-2 is expressed as:

$$\tau = \frac{c' + (\sigma - u)\tan\phi'}{F} \qquad \text{(C-5)}$$

The factor of safety is computed by repeatedly assuming values for F and calculating the corresponding shear stress from Equation C-5 until equilibrium is achieved. In effect, the strength is reduced by the factor of safety, F, until a just-stable, or limiting, equilibrium condition is achieved. Equation C-5 can be expanded and written as:

$$\tau = \frac{c'}{F} + \frac{(\sigma - u)\tan\phi'}{F} \qquad \text{(C-6)}$$

The first term represents the contribution of "cohesion" to shear resistance; the second term represents the contribution of "friction." The "developed" cohesion and friction are defined as follows:

$$c'_D = \frac{c'}{F} \qquad \text{(C-7)}$$

and

$$\tan\phi'_D = \frac{\tan\phi'}{F} \qquad \text{(C-8)}$$

where

$c'_D$ = developed cohesion

$\phi'_D$ = developed friction angle

*d. Assumptions in methods of slices.* Many of the limit equilibrium methods (Ordinary Method of Slices (OMS), Simplified Bishop, Corps of Engineers' Modified Swedish, Spencer) address static equilibrium by dividing the soil mass above the assumed slip surface into a finite number of vertical slices. The forces acting on an individual slice are illustrated in Figure C-2. The forces include:

W - slice weight

E - horizontal (normal) forces on the sides of the slice

X - vertical (shear) forces between slices

N - normal force on the bottom of the slice

S - shear force on the bottom of the slice

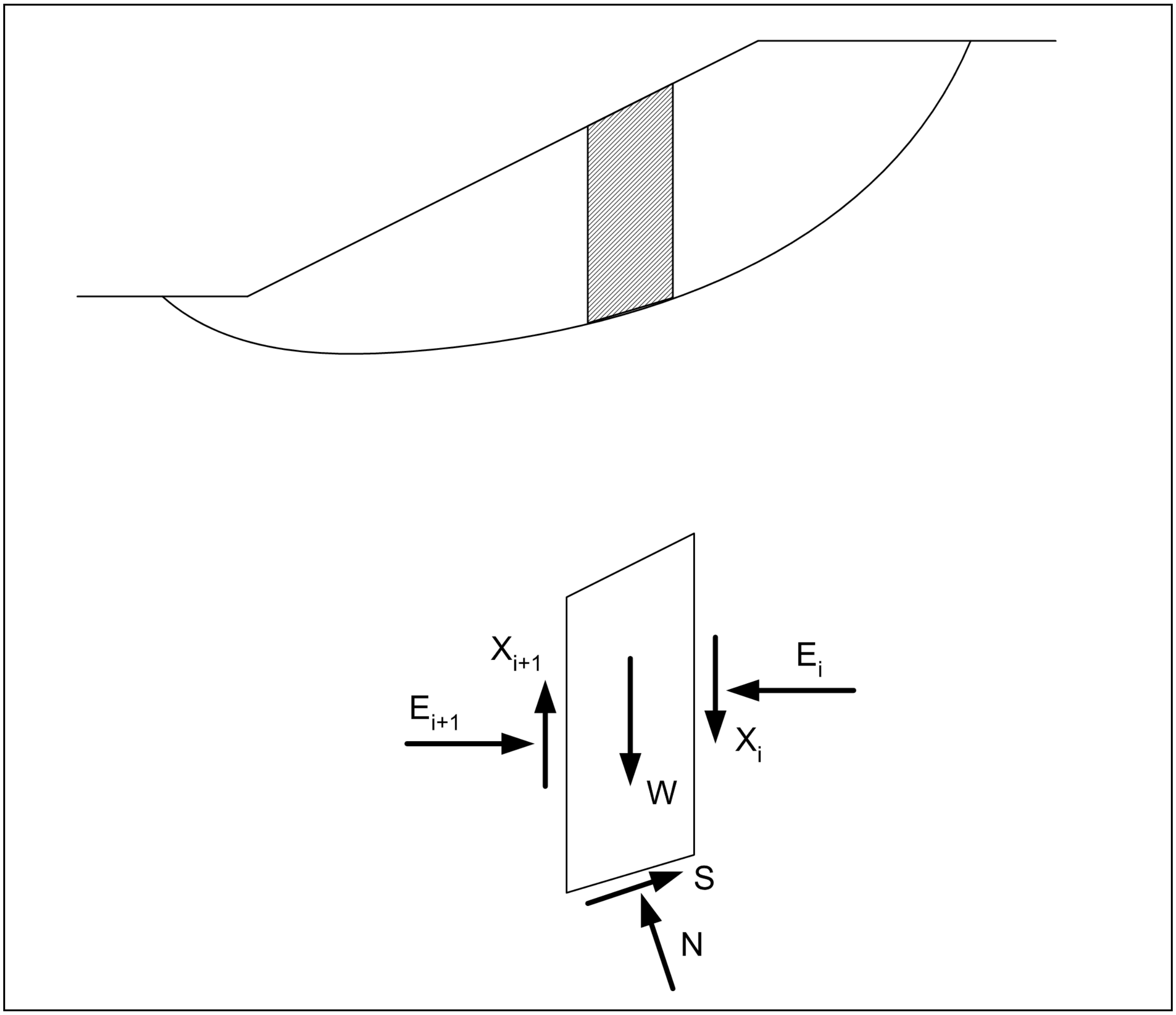



**Figure C-2. Typical slice and forces for method of slices**

Except for the weight of the slice, all of these forces are unknown and must be calculated in a way that satisfies static equilibrium.

(1) For the current discussion, the shear force (S) on the bottom of the slice is not considered directly as an unknown in the equilibrium equations that are solved. Instead, the shear force is expressed in terms of other known and unknown quantities, as follows: S on the base of a slice is equal to the shear stress, $\tau$, multiplied by the length of the base of the slice, $\Delta\ell$, i.e.,

$$S = \tau\Delta\ell \tag{C-9}$$

or, by introducing Equation C-5, which is based on the definition of the factor of safety,

$$S = \frac{c'\Delta\ell}{F} + \frac{(\sigma - u)\Delta\ell \tan\phi'}{F} \tag{C-10}$$

Finally, noting that the normal force N is equal to the product of the normal stress ($\sigma$) and the length of the bottom of the slice ($\Delta\ell$), i.e., N = $\sigma\,\Delta\ell$, Equation C-9 can be written as:

$$S = \frac{c'\Delta\ell}{F} + \frac{(N - u\Delta\ell)\tan\phi'}{F} \tag{C-11}$$

(2) Equation C-11 relates the shear force, S, to the normal force on the bottom of the slice and the factor of safety. Thus, if the normal force and factor of safety can be calculated from the equations of static equilibrium, the shear force can be calculated (is known) from Equation C-11. Equation C-11 is derived from the Mohr-Coulomb equation and the definition of the factor of safety, independently of the conditions of static equilibrium. The forces and other unknowns that must be calculated from the equilibrium equations are summarized in Table C-1. As discussed above, the shear force, S, is not included in Table C-1, because it can be calculated from the unknowns listed and the Mohr-Coulomb equation (C-11), independently of static equilibrium equations.

**Table C-1**
**Unknowns and Equations for Limit Equilibrium Methods**

| **Unknowns** | **Number of Unknowns for n Slices** |
|---|---|
| Factor of safety (F) | 1 |
| Normal forces on bottom of slices (N) | N |
| Interslice normal forces, E | n – 1 |
| Interslice shear forces, X | n – 1 |
| Location of normal forces on base of slice | N |
| Location of interslice normal forces | n – 1 |
| TOTAL NUMBER OF UNKNOWNS | 5n – 2 |
| **Equations** | **Number of Equations for n Slices** |
| Equilibrium of forces in the horizontal direction, $\Sigma F_x = 0$ | n |
| Equilibrium of forces in the vertical direction, $\Sigma F_y = 0$ | n |
| Equilibrium of moments | n |
| TOTAL NUMBER OF EQUILIBRIUM EQUATIONS | 3n |

(3) In order to achieve a statically determinate solution, there must be a balance between the number of unknowns and the number of equilibrium equations. The number of equilibrium equations is shown in the lower part of Table C-1. The number of unknowns (5n – 2) exceeds the number of equilibrium equations (3n) if n is greater than one. Therefore, some assumptions must be made to achieve a statically determinate solution.

(4) The various limit equilibrium methods use different assumptions to make the number of equations equal to the number of unknowns. They also differ with regard to which equilibrium equations are satisfied. For example, the Ordinary Method of Slices, the Simplified Bishop Method, and the U.S. Army Corps of Engineers' Modified Swedish Methods do not satisfy all the conditions of static equilibrium. Methods such as the Morgenstern and Price's and Spencer's do satisfy all static equilibrium conditions. Methods that satisfy static equilibrium fully are referred to as "complete" equilibrium methods. Details of various limit equilibrium procedures and their differences are presented in Sections C-2 through C-7. Detailed comparison of limit equilibrium slope stability analysis methods have been reported by Whitman and Bailey (1967), Wright (1969), Duncan and Wright (1980) and Fredlund and Krahn (1977).[1]

*e. Limitations of limit equilibrium methods.* Complete equilibrium methods have generally been more accurate than those procedures which do not satisfy complete static equilibrium and are therefore preferable to

[1] References information is presented in Appendix A.

"incomplete" methods. However, the "incomplete" methods are often sufficiently accurate and useful for many practical applications, including hand checks and preliminary analyses. In all of the procedures described in this manual, the factor of safety is applied to both cohesion and friction, as shown by Equation C-6.

(1) The factor of safety is also assumed to be constant along the shear surface. Although the factor of safety may not in fact be the same at all points on the slip surface, the average value computed by assuming that F is constant provides a valid measure of stability for slopes in ductile (nonbrittle) soils. For slopes in brittle soils, the factor of safety computed assuming F is the same at all points on the slip surface may be higher than the actual factor of safety.

(2) If the strength is fully mobilized at any point on the slip surface, the soil fails locally. If the soil has brittle stress-strain characteristics so that the strength drops once the peak strength is mobilized, the stress at that point of failure is reduced and stresses are transferred to adjacent points, which in turn may then fail. In extreme cases this may lead to progressive failure and collapse of the slope. If soils possess brittle stress-strain characteristics with relatively low residual shear strengths compared to the peak strengths, reduced strengths and/or higher factors of safety may be required for stability. Limitations of limit equilibrium procedures are summarized in Table C-2.

**Table C-2**
**Limitations of Limit-Equilibrium Methods**

| |
|---|
| 1. The factor of safety is assumed to be constant along the potential slip surface. |
| 2. Load-deformation (stress-strain) characteristics are not explicitly accounted for. |
| 3. The initial stress distribution within the slope is not explicitly accounted for. |
| 4. Unreasonably large and or negative normal forces may be calculated along the base of slices under certain conditions (SectionC-l0.b and C-10.c). |
| 5. Iterative, trial and error, solutions may not converge in certain cases (Section C-10d). |

*f. Shape of the slip surface.* All of the limit equilibrium methods require that a potential slip surface be assumed in order to calculate the factor of safety. Calculations are repeated for a sufficient number of trial slip surfaces to ensure that the minimum factor of safety has been calculated. For computational simplicity the candidate slip surface is often assumed to be circular or composed of a few straight lines (Figure C-3). However, the slip surface will need to have a more complicated shape in complex stratigraphy. The assumed shape is dependent on the problem geometry and stratigraphy, material characteristics (especially anisotropy), and the capabilities of the analysis procedure used. Commonly assumed shapes are discussed below.

(1) Circular. Observed failures in relatively homogeneous materials often occur along curved failure surfaces. A circular slip surface, like that shown in Figure C-3a, is often used because it is convenient to sum moments about the center of the circle, and because using a circle simplifies the calculations. A circular slip surface must be used in the Ordinary Method of Slices and Simplified Bishop Method. Circular slip surfaces are almost always useful for starting an analysis. Also, circular slip surfaces are generally sufficient for analyzing relatively homogeneous embankments or slopes and embankments on foundations with relatively thick soil layers.

(2) Wedge. "Wedge" failure mechanisms are defined by three straight line segments defining an active wedge, central block, and passive wedge (Figure C-3b). This type of slip surface may be appropriate for slopes where the critical potential slip surface includes a relatively long linear segment through a weak material bounded by stronger material. A common example is a relatively strong levee embankment founded on weaker, stratified alluvial soils. Wedge methods, including methods for defining or calculating the inclination of the base of the wedges, are discussed in Section C-1g.

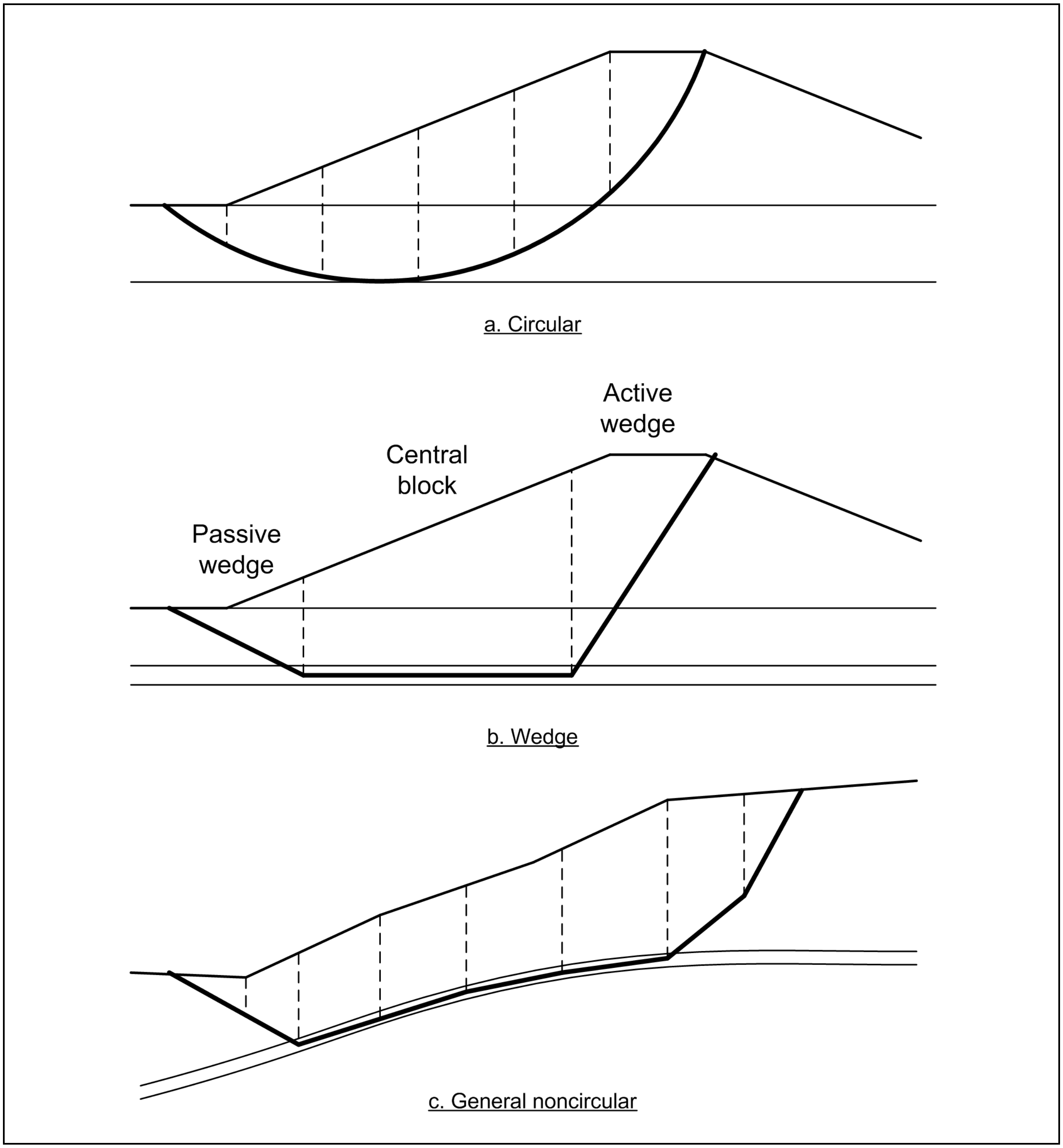



**Figure C-3. Shapes for potential slip surfaces**

(3) Two circular segments with a linear midsection. This is a combination of the two shapes (circular and wedge) discussed above that is used by some computer programs.

(4) General, noncircular shape. Slope failure may occur by sliding along surfaces that do not correspond to either the wedge or circular shapes. The term general slip surface refers to a slip surface composed of a number of linear segments which may each be of any length and inclined at any angle. The term "noncircular" is also used in reference to such general-shaped slip surfaces. Prior to about 1990, slip surfaces of a general shape, other than simple wedges, were seldom analyzed, largely because of the difficulty in

systematically searching for the critical slip surface. However, in recent years improved search techniques and computer software have increased the capability to analyze such slip surfaces. Stability analyses based on general slip surfaces are now much more common and are useful as a design check of critical slip surfaces of traditional shapes (circular, wedge) and where complicated geometry and material conditions exist. It is especially important to investigate stability with noncircular slip surfaces when soil shear strengths are anisotropic.

(a) Inappropriate selection of the shape for the slip surface can cause computational difficulties and erroneous solutions.

(b) A common problem occurs near the toe of the slope when the slip surface exits too steeply through materials with large values of $\phi$ or $\phi'$.

(c) The problem of a steeply exiting slip surface is especially important and is covered in further detail in Section C-10.b.

*g. Location of the critical slip surface.* The critical slip surface is defined as the surface with the lowest factor of safety. Because different analysis procedures employ different assumptions, the location of the critical slip surface can vary somewhat among different methods of analysis. The critical slip surface for a given problem analyzed by a given method is found by a systematic procedure of generating trial slip surfaces until the one with the minimum factor of safety is found. Searching schemes vary with the assumed shape of the slip surface and the computer program used. Common schemes are discussed below.

(1) Circular slip surfaces. Search schemes for circular arc slip surfaces are illustrated in Figures C-4, C-5, and C-6. A circular surface is defined by the position of the circle center and either (a) the radius, (b) a point through which the circle must pass, or (c) a plane to which the slip surface must be tangent. In case (b), the toe of the slope is often specified as the point through which the circle must pass. Searches are usually accomplished by changing one of these variables and varying a second variable until a minimum factor of safety is found. For example, the location of the center point may be varied while the plane of tangency is fixed, or the radius may be varied while the center point is fixed. The first search variable is then fixed at a new value and the second variable is again varied. This process is repeated until the minimum factor of safety corresponding to both search variables is found. For a homogeneous slope in cohesionless soil ($c = 0$, $c' = 0$), a critical circle will degenerate to a plane parallel to the slope and the factor of safety will be identical to the one for an infinite slope. Theoretically, the critical "circle" will be one having a center point located an infinite distance away from the slope on a line perpendicular to the midpoint of the slope. The circle will have an infinite radius as well. When attempts are made to search for a critical circle in a homogeneous slope of cohesionless soil with most computer programs, the search will appear to "run-away" from the slope. The search will probably be stopped eventually as a result of either numerical errors and roundoff or some constraint imposed by the software being used. In such cases the Infinite Slope analysis procedure (Section C-7) should be used.

(2) Wedge-shaped slip surfaces. Wedge-shaped slip surfaces require searching for the critical location of the central block and for the critical inclination of the bases of the active and passive wedges. Searching for the critical location of the central block is illustrated in Figure C-7a and involves systematically varying the horizontal and vertical coordinates of the two ends of the base of the central block, until the central block corresponding to the minimum factor of safety is found. For each trial position of the central block, the base inclinations of the active and passive wedge segments must be set based on simple rules or by searching to locate critical inclinations. A simple and common assumption is to make the inclination of each active wedge segment (measured from the horizontal) $45 + \phi'_D/2$ degrees, and of each passive wedge segment $45 - \phi'_D/2$ degrees. The quantity $\phi'_D$ represents the developed friction angle ($\tan \phi'_D = \tan \phi'/F$) and should be

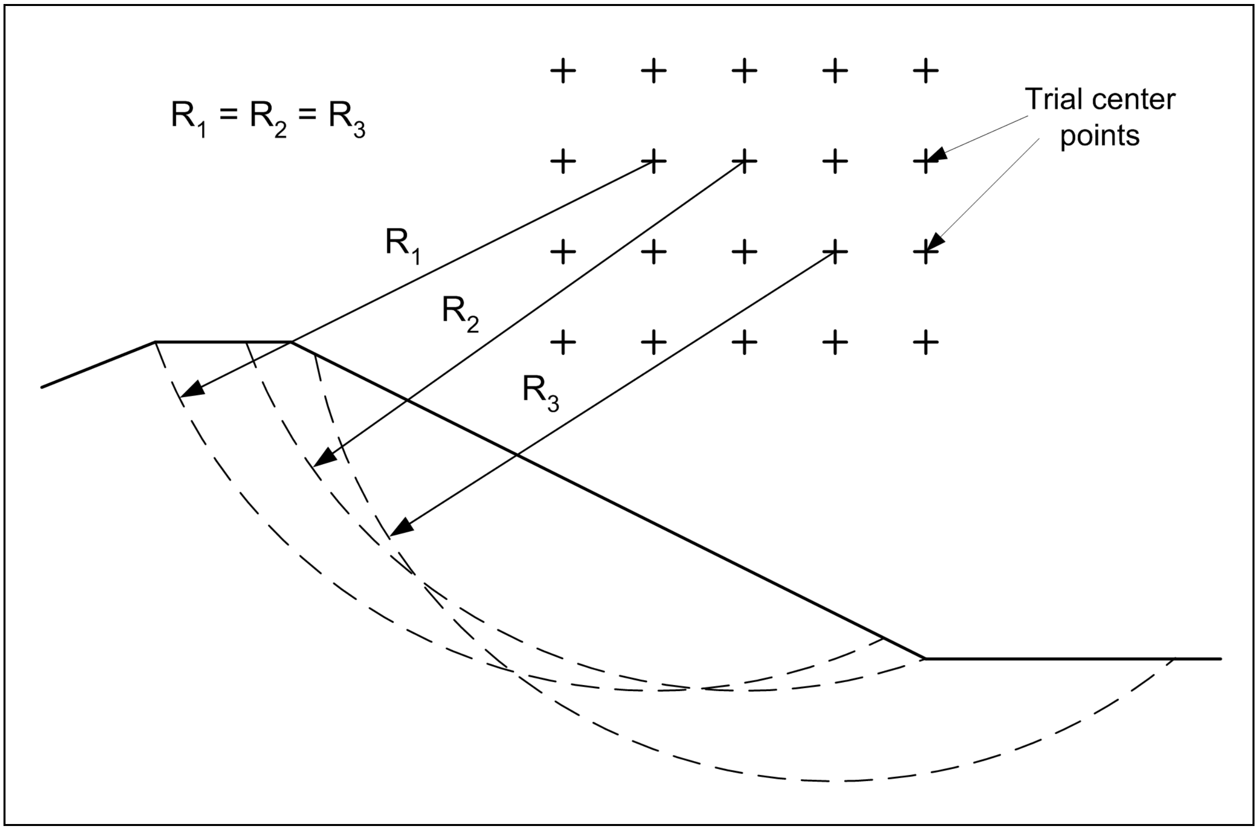



**Figure C-4. Search with constant radius**

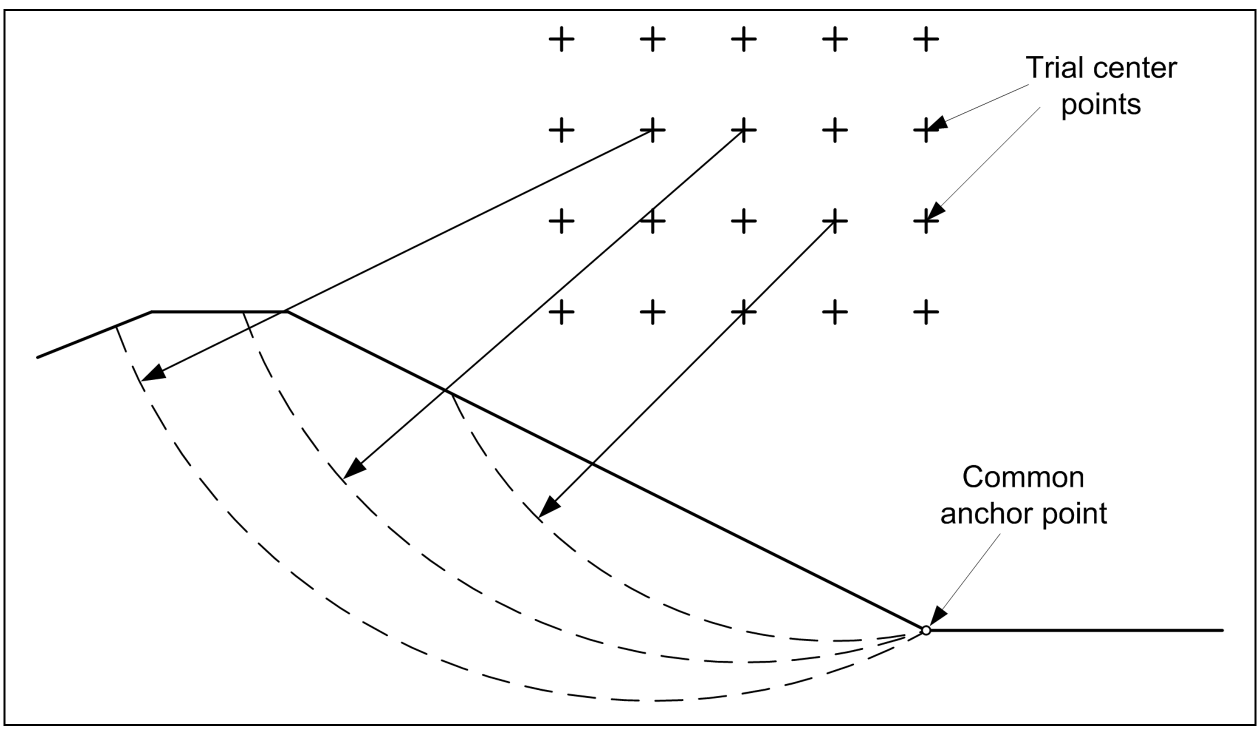



**Figure C-5. Search with circles through a common point**

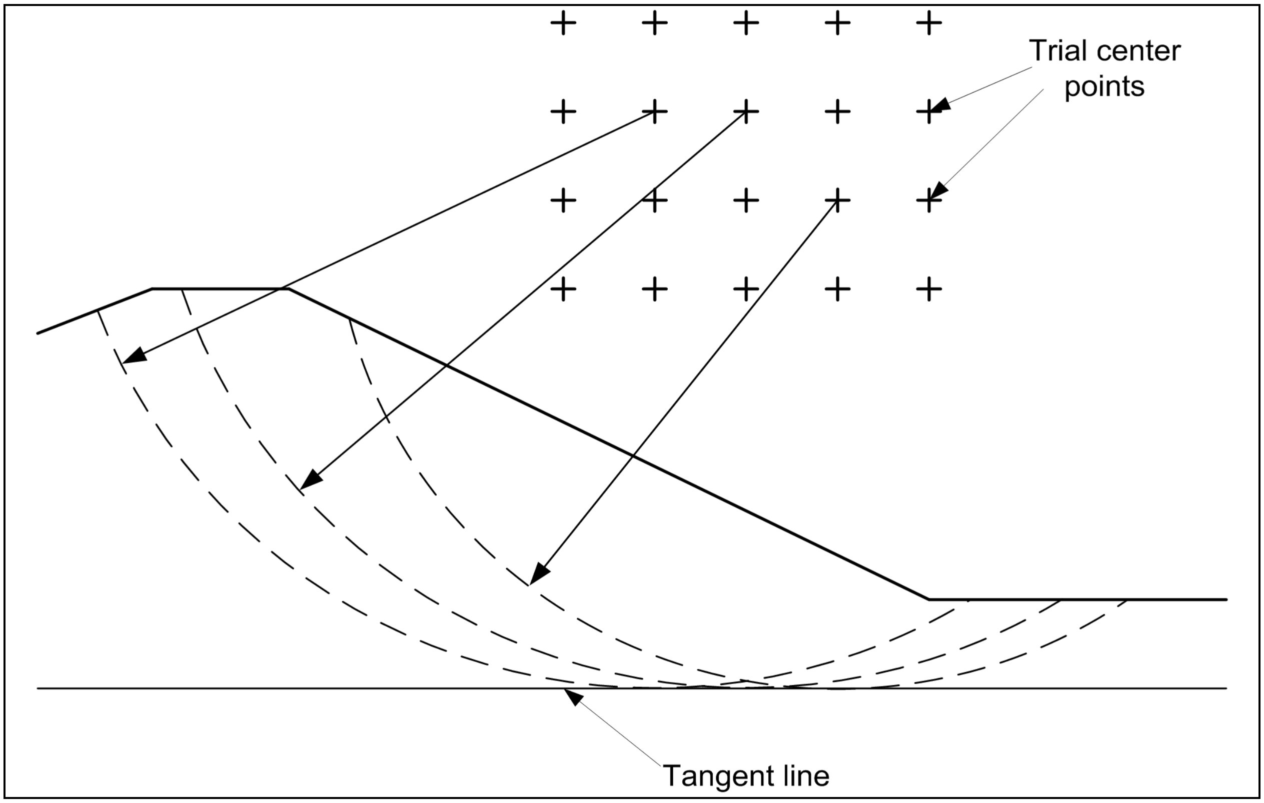



**Figure C-6. Search with circles tangent to a prescribed tangent line**

consistent with the computed factor of safety. This assumption for the inclination of the active and passive wedges is only appropriate where the top surfaces of the active and passive wedges are horizontal but provides reasonable results for gently inclined slopes. Common methods for searching for the inclination of the base of the wedges are shown in Figure C-7b. One technique, used where soil properties and inclinations of the base of each wedge vary in the zone of the active and passive wedges, is to assume that the bottoms of the wedges are inclined at $\alpha = \theta \pm \phi'_D/2$. The value of $\theta$ is then varied until the maximum interslice force is found for the active wedge and minimum interslice force is found for the passive wedge. A second search technique, where the bases of the active and passive wedges are considered to be single planes, is to vary the value of $\alpha$ until a maximum interslice force is obtained for the entire group of active wedge segments and the minimum is found for the entire group of passive wedge segments.

(3) General shapes. A number of techniques have been proposed and used to locate the most critical general-shaped slip surface. One of the most robust and useful procedures is the one developed by Celestino and Duncan (1981). The method is illustrated in Figure C-8. In this method, an initial slip surface is assumed and represented by a series of points that are connected by straight lines. The factor of safety is first calculated for the assumed slip surface. Next, all points except one are held fixed, and the “floating” point is shifted a small distance in two directions. The directions might be vertically up and down, horizontally left and right, or above and below the slip surface in some assumed direction. The factor of safety is calculated for the slip surface with each point shifted as described. This process is repeated for each point on the slip surface. As any one point is shifted, all other points are left at their original location. Once all points have been shifted in both directions and the factor of safety has been computed for each shift, a new location is estimated for the slip surface based on the computed factors of safety. The slip surface is then moved to the estimated location and the process of shifting points is repeated. This process is continued until no further reduction in factor of safety is noted and the distance that the shear surface is moved on successive approximations becomes minimal.

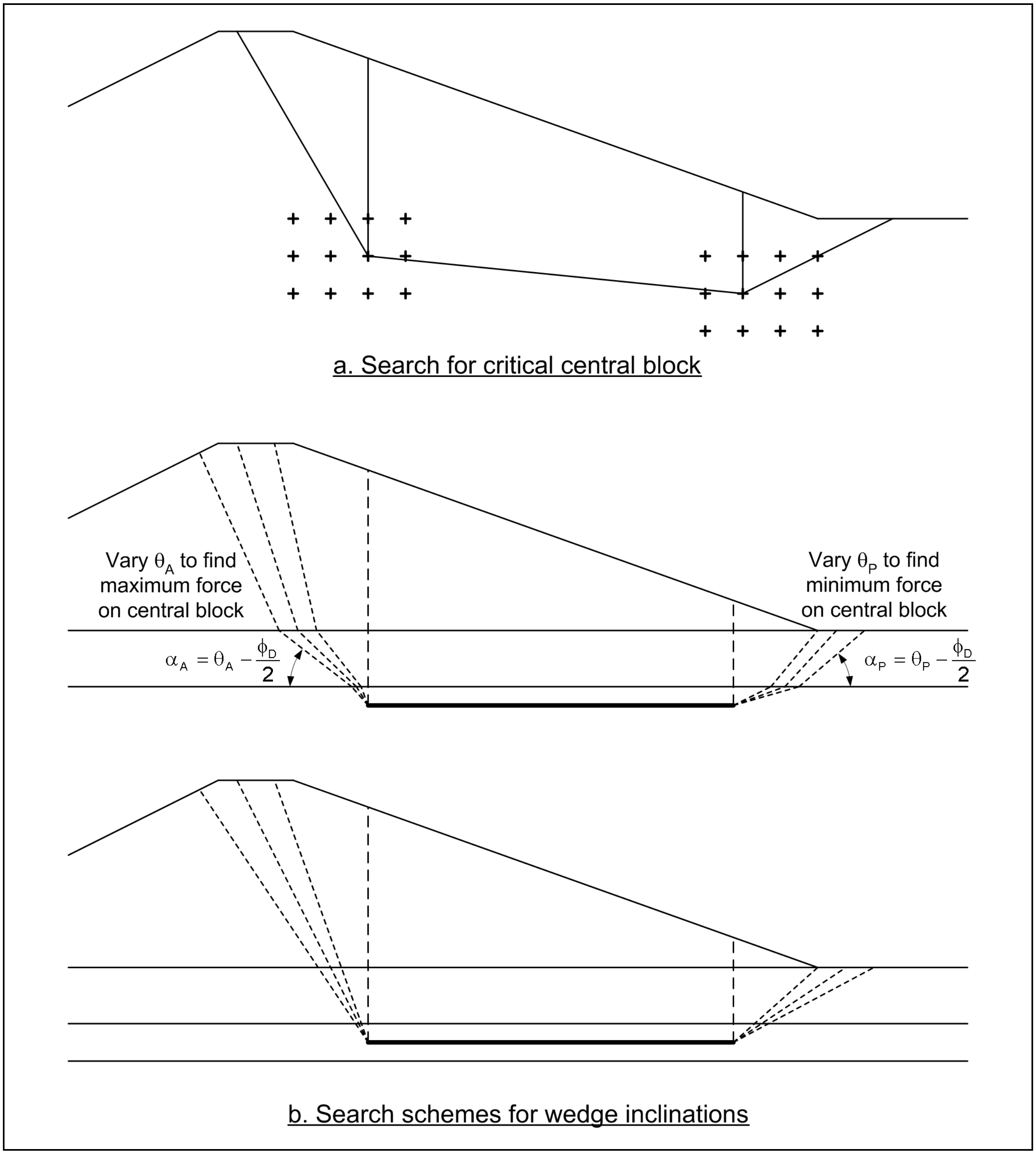



**Figure C-7. Search schemes for wedges**

(4) Limitations and precautions. Any search scheme employed in computer programs is restricted to investigating a finite number of slip surfaces. In addition, most of these schemes are designed to locate one slip surface with a minimum factor of safety. The schemes may not be able to locate more than one local minimum. The results of automatic searches are dependent on the starting location for the search and any constraints that are imposed on how the slip surface is moved. Automatic searches are controlled largely by the data that the user inputs into the software. Regardless of the software used, a number of separate searches should be conducted to confirm that the lowest factor of safety has been calculated.

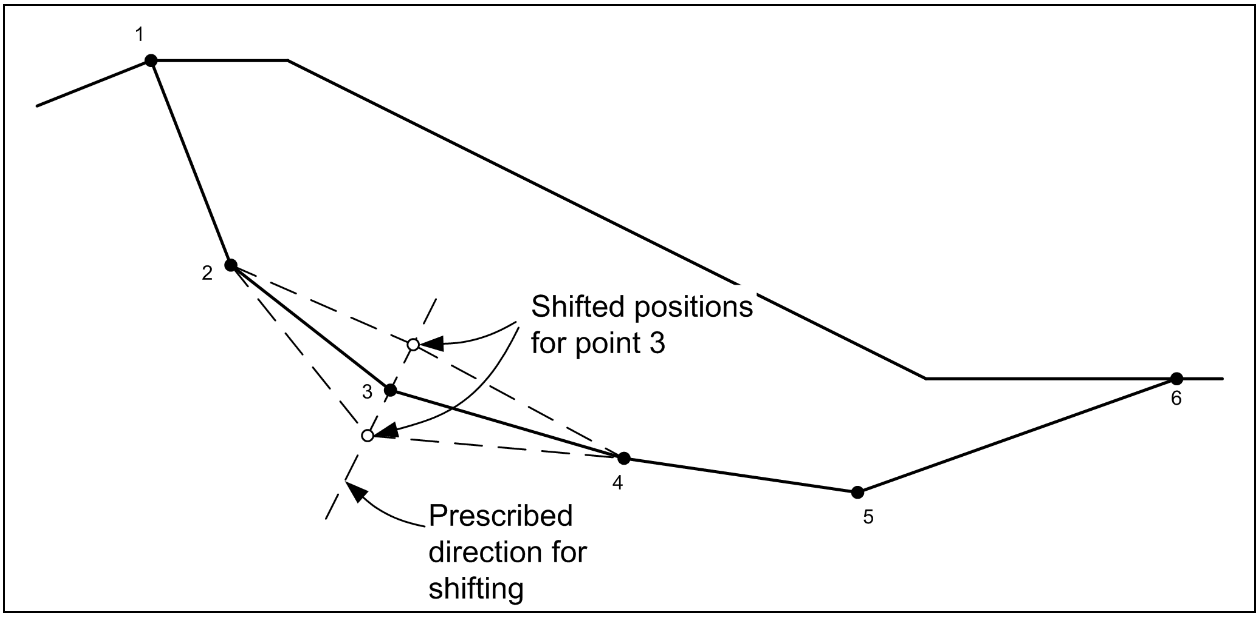



**Figure C-8. Search scheme for noncircular slip surfaces (after Celistono and Duncan 1981)**

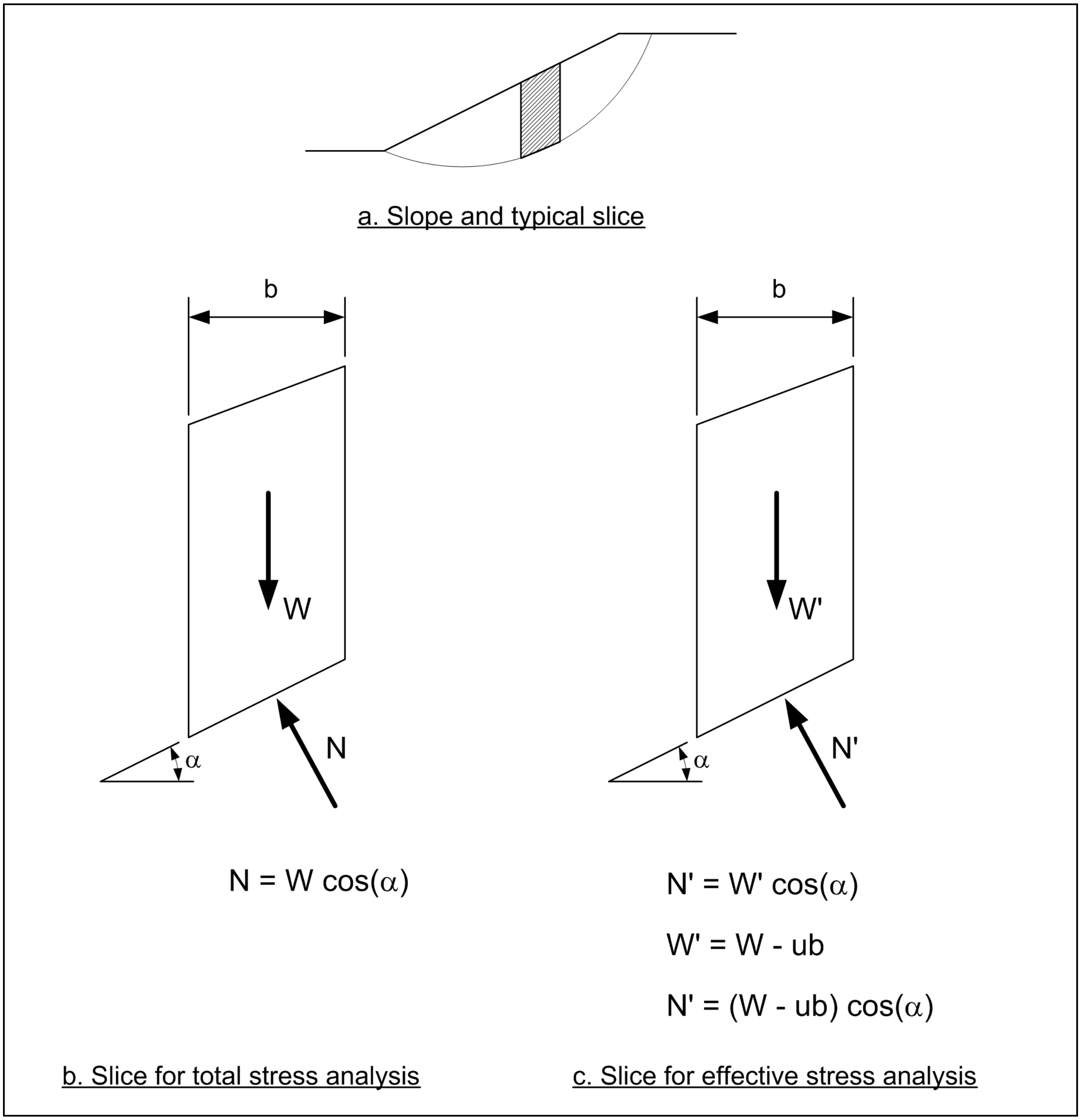
(a) In some cases it is appropriate to calculate the factor of safety for selected potential slip surfaces that do not necessarily produce the minimum factor of safety but would be more significant in terms of the consequences of failure. For example, in slopes that contain cohesionless soil at the face of the slope, the lowest factor of safety may be found for very shallow (infinite slope) slip surfaces, yet shallow sloughing is usually much less important than deeper-seated sliding.

(b) Mine tailings, disposal dams, and cohesionless fill slopes on soft clay foundations provide examples where deeper slip surfaces than the one producing the minimum factor of safety are often more important. In such cases, deeper slip surfaces should be investigated in addition to the shallow slip surfaces having the lowest factors of safety.

*h. Probabilistic methods.* Conventional slope stability analyses are deterministic methods; meaning that all variables are assumed to have specific values. Probabilistic methods consider uncertainties in the values of the variables and evaluate the effects of these uncertainties on the computed values of factor of safety. Probabilistic approaches can be used in conjunction with any of the limit equilibrium stability methods. ETL 1110-2-556 (1999) describes techniques for probabilistic analyses and their application to slope stability studies.

## C-2. The Ordinary Method of Slices

*a. Assumptions.* The Ordinary Method of Slices (OMS) was developed by Fellenius (1936) and is sometimes referred to as "Fellenius' Method." In this method, the forces on the sides of the slice are neglected (Figure C-9). The normal force on the base of the slice is calculated by summing forces in a direction perpendicular to the bottom of the slice. Once the normal force is calculated, moments are summed about the center of the circle to compute the factor of safety. For a slice and the forces shown in Figure C-9, the factor of safety is computed from the equation,




a. Slope and typical slice

b. Slice for total stress analysis

c. Slice for effective stress analysis

**Figure C-9. Typical slice and forces for Ordinary Method of Slices**

$$F = \frac{\sum\left[c'\Delta\ell + \left(W\cos\alpha - u\Delta\ell\cos^2\alpha\right)\tan\phi'\right]}{\sum W\sin\alpha} \tag{C-12}$$

where

$c'$ and $\phi'$ = shear strength parameters for the center of the base of the slice

$W$ = weight of the slice

$\alpha$ = inclination of the bottom of the slice

u = pore water pressure at the center of the base of the slice

$\Delta\ell$ = length of the bottom of the slice

As shown in Table C-3, there is only one unknown in the Ordinary Method of Slices (F), and only one equilibrium equation is used (the equation of equilibrium of the entire soil mass around the center of the circle).

**Table C-3**
**Unknowns and Equations for the Ordinary Method of Slices Procedure**

| Unknowns | Number of Unknowns for n Slices |
|---|---|
| Factor of safety (F) | 1 |
| TOTAL NUMBER OF UNKNOWNS | 1 |
| **Equations** | **Number of Equations for n Slices** |
| Equilibrium of moments of the entire soil mass | 1 |
| TOTAL NUMBER OF EQUILIBRIUM EQUATIONS | 1 |

(1) Two different equations have been used to compute the factor of safety by the OMS with effective stresses and pore water pressures. The first equation is shown above as Equation C-12. Equation C-12 is derived by first calculating an "effective" slice weight, W', by subtracting the uplift force due to pore water pressure from the weight, and then resolving forces in a direction perpendicular to the base of the slice (Figure C-9). The other OMS equation for effective stress analyses is written as:

$$F = \frac{\sum\left[c'\Delta\ell + \left(W\cos\alpha - u\Delta\ell\right)\tan\phi'\right]}{\sum W\sin\alpha} \tag{C-13}$$

Equation C-13 is derived by first resolving the force because of the total slice weight (W) in a direction perpendicular to the base of the slice and then subtracting the force because of pore water pressures. Equation C-12 leads to more reasonable results when pore water pressures are used. Equation C-13 can lead to unrealistically low or negative stresses on the base of the slice because of pore water pressures and should not be used.

(2) External water on a slope can be treated in either of two ways: The water may simply be represented as soil with c = 0 and $\phi$ = 0. In this case, the trial slip surface is assumed to extend through the water and exit at the surface of the water. Some of the slices will then include water and the shear strength for any slices whose base lies in water will be assigned as zero. The second way that water can be treated in an analysis is to treat the water as an external, hydrostatic load on the top of the slices. In this case, the trial slip surface will only pass through soil, and each end will exit at the ground or slope surface (Figure C-10). For the equations presented in this appendix as well as the examples in Appendixes F and G, the water is treated as an external load. Treating the water as another "soil" involves simply modifying the geometry and properties of the slices.

(3) In the case where water loads act on the top of the slice, the expression for the factor of safety (Equation C-12) must be modified to the following:

$$F = \frac{\sum\left\{c'\Delta\ell + \left[W\cos\alpha + P\cos\left(\alpha-\beta\right) - u\Delta\ell\cos^2\alpha\right]\tan\phi'\right\}}{\sum W\sin\alpha - \frac{\sum M_P}{R}} \tag{C-14}$$

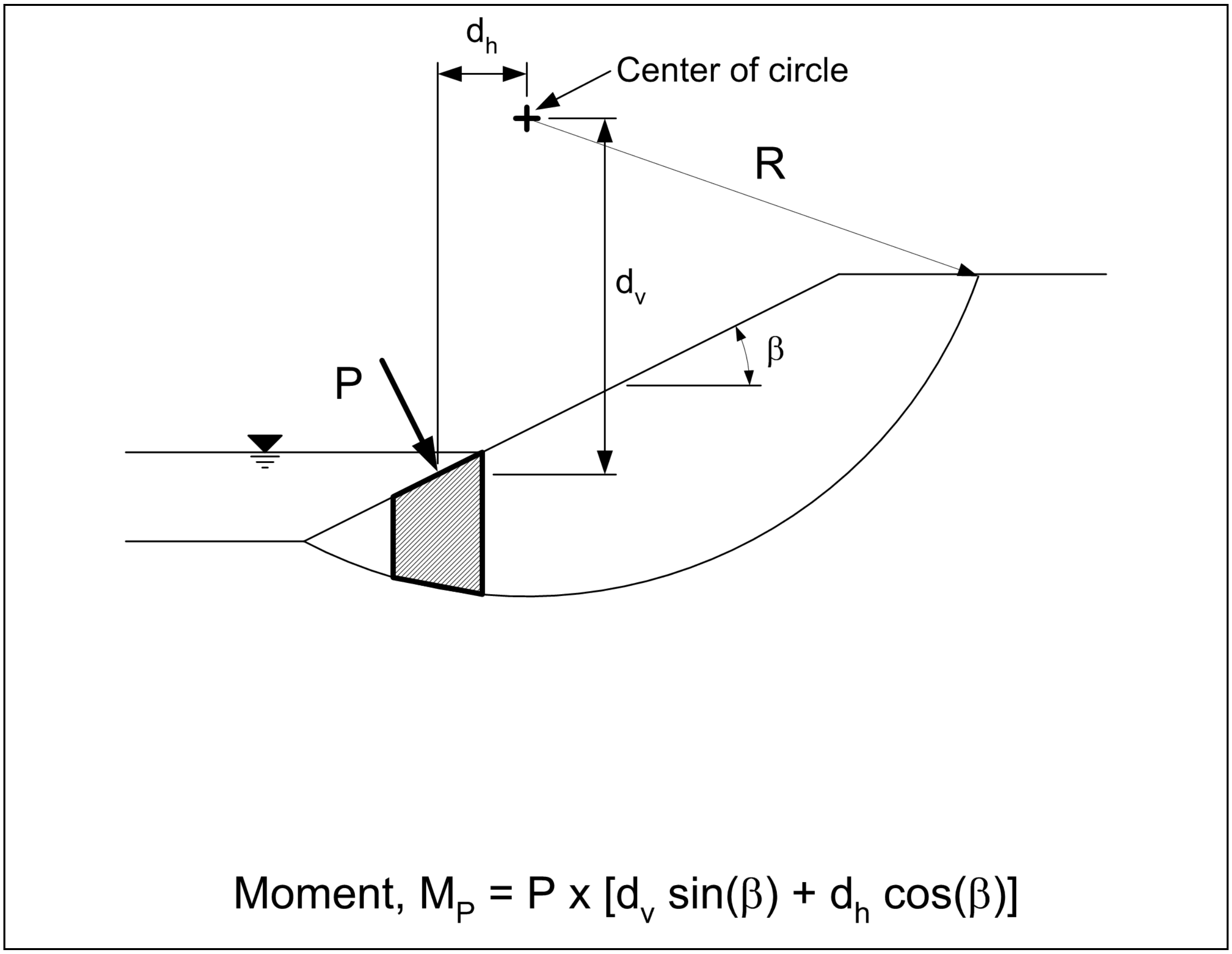



**Figure C-10. Slice for Ordinary Method of Slices with external water loads**

where

$P$ = resultant water force acting perpendicular to the top of the slice

$\beta$ = inclination of the top of the slice

$M_P$ = moment about the center of the circle produced by the water force acting on the top of the slice

$R$ = radius of the circle (Figure C-10).

The moment, $M_P$, is considered to be positive when it acts in the opposite direction to the moment produced by the weight of the sliding mass.

*b. Limitations.* The principal limitation of the OMS comes from neglecting the forces on the sides of the slice. The method also does not satisfy equilibrium of forces in either the vertical or horizontal directions. Moment equilibrium is satisfied for the entire soil mass above the slip surface, but not for individual slices.

(1) Factors of safety calculated by the OMS may commonly differ as much as 20 percent from values calculated using rigorous methods (Whitman and Bailey 1967); in extreme cases (such as effective stress analysis with high pore water pressures), the differences may be even larger. The error is generally on the safe side (calculated factor of safety is too low), but the error may be so large as to yield uneconomical designs. Because of the tendency for errors to be on the "safe side," the OMS is sometimes mistakenly thought always to produce conservative values for the factor of safety. This is not correct. When $\phi = 0$, the OMS yields the same factor of safety as more rigorous procedures, which fully satisfy static equilibrium. Thus, the degree to which the OMS is conservative depends on the value of $\phi$ and whether the pore pressures are large or small.

(2) Although Equation C-12 does not specifically include the radius of the circle, the equation is based on the assumption that the slip surface is circular. The OMS can only be used with circular slip surfaces.

*c. Recommendation for use.* The OMS is included herein for reference purposes and completeness because numerous existing slopes have been designed using the method. As the method still finds occasional use in practice, occasions may arise where there is a need to review designs by others that were based on the method. Also, because the OMS is simple, it is useful where calculations must be done by hand using an electronic calculator. The method also may be used to overcome problems that may develop near the toe of steeply exiting shear surfaces as described in Section C-10.b.

## C-3. The Simplified Bishop Method

*a. Assumptions.* The Simplified Bishop Method was developed by Bishop (1955). This procedure is based on the assumption that the interslice forces are horizontal, as shown in Figure C-11. A circular slip surface is also assumed in the Simplified Bishop Method. Forces are summed in the vertical direction. The resulting equilibrium equation is combined with the Mohr-Coulomb equation and the definition of the factor of safety to determine the forces on the base of the slice. Finally, moments are summed about the center of the circular slip surface to obtain the following expression for the factor of safety:

$$F = \frac{\sum \left[ \frac{c'\Delta x + \left( W + P\cos\beta - u\Delta x \sec\alpha \right) \tan\phi'}{m_\alpha} \right]}{\sum W \sin\alpha - \frac{\sum M_P}{R}} \tag{C-15}$$

where $\Delta x$ is the width of the slice, and $m_\alpha$ is defined by the following equation,

$$m_\alpha = \cos\alpha + \frac{\sin\alpha \tan\phi'}{F} \tag{C-16}$$

The terms W, c', $\phi'$, u, P, $M_P$, and R are as defined earlier for the OMS. Factors of safety calculated from Equation C-15 satisfy equilibrium of forces in the vertical direction and overall equilibrium of moments about the center of a circle. The unknowns and equations in the Simplified Bishop Method are summarized in Table C-4.

Because the value of the term $m_\alpha$ depends on the factor of safety, the factor of safety appears on both sides of Equation C-15. Equation C-15 cannot be manipulated such that an explicit expression is obtained for the factor of safety. Thus, an iterative, trial and error procedure is used to solve for the factor of safety.

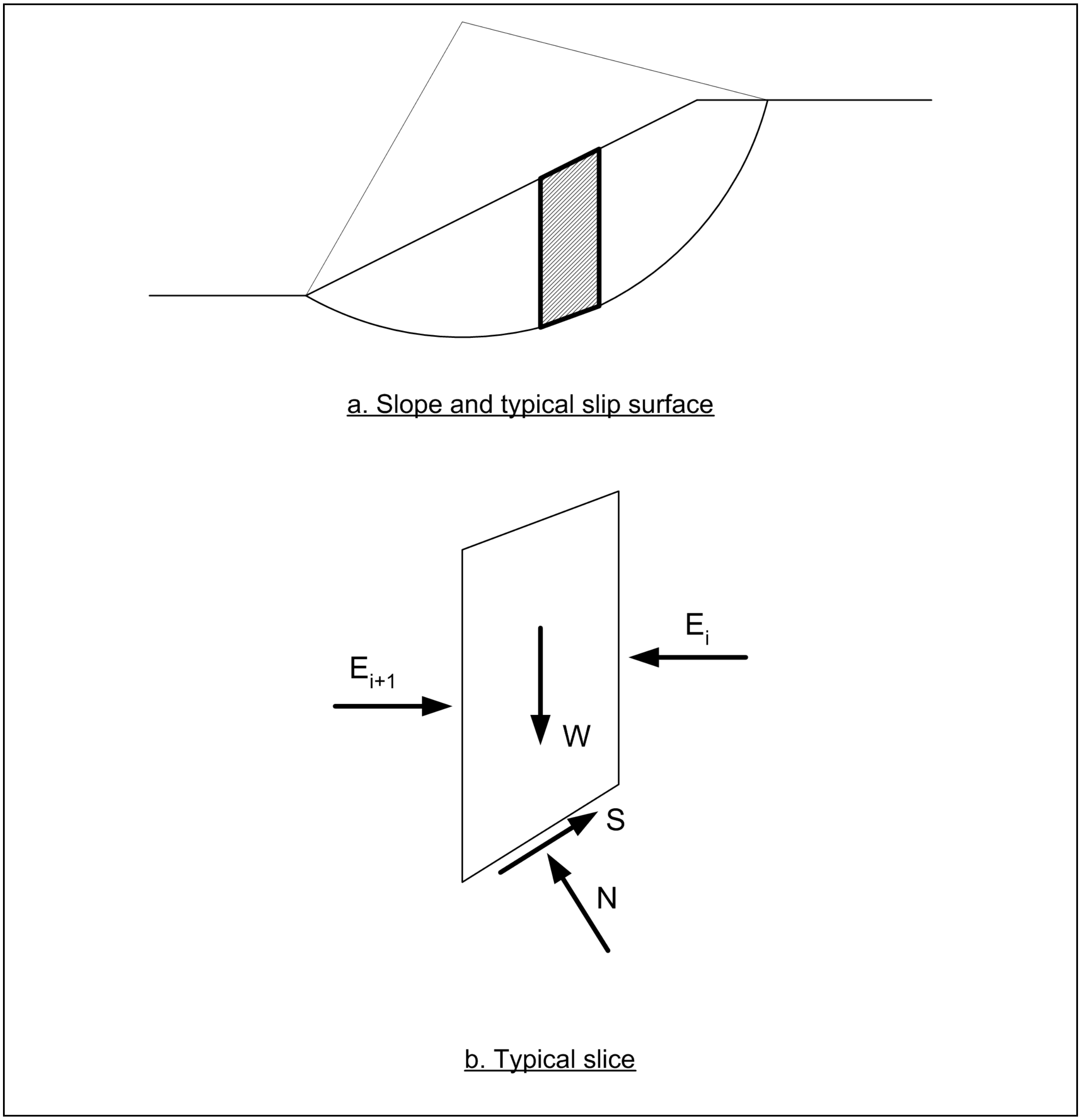



**Figure C-11. Typical slice and forces for Simplified Bishop Method**

**Table C-4**
**Unknowns and Equations for the Simplified Bishop Method**

| **Unknowns** | **Number of Unknowns for n Slices** |
|---|---|
| Factor of safety (F) | 1 |
| Normal forces on bottom of slices (N) | n |
| TOTAL NUMBER OF UNKNOWNS | n + 1 |
| **Equations** | **Number of Equations for n Slices** |
| Equilibrium of forces in the vertical direction, $\Sigma F_y = 0$ | n |
| Equilibrium of moments of the entire soil mass | 1 |
| TOTAL NUMBER OF EQUILIBRIUM EQUATIONS | n + 1 |

*b. Limitations.* Horizontal equilibrium of forces is not satisfied by the Simplified Bishop Method. Because horizontal force equilibrium is not completely satisfied, the suitability of the Simplified Bishop Method for pseudo-static earthquake analyses where an additional horizontal force is applied is questionable. The method is also restricted to analyses with circular shear surfaces.

*c. Recommendation for use.* It has been shown by a number of investigators (Whitman and Bailey 1967; Fredlund and Krahn 1977) that the factors of safety calculated by the Simplified Bishop Method compare well with factors of safety calculated using rigorous methods, usually within 5 percent. Furthermore, the procedure is relatively simple compared to more rigorous solutions, computer solutions execute rapidly, and hand calculations are not very time-consuming. The method is widely used throughout the world, and thus, a strong record of experience with the method exists. The Simplified Bishop Method is an acceptable method of calculating factors of safety for circular slip surfaces. It is recommended that, where major structures are designed using the Simplified Bishop Method, the final design should be checked using Spencer's Method.

*d. Verification procedures.* When the Simplified Bishop Method is used for computer calculations, results can be verified by hand calculations using a calculator or a spreadsheet program, or using slope stability charts. An approximate check of calculations can also be performed using the Ordinary Method of Slices, although the OMS will usually give a lower value for the factor of safety, especially if $\phi$ is greater than zero and pore pressures are high.

## C-4. Force Equilibrium Method, Including the Modified Swedish Method

*a. Assumptions.* Force equilibrium methods satisfy force equilibrium in both the horizontal and vertical directions, but they do not satisfy moment equilibrium. All force equilibrium methods are based on assuming the inclinations ($\theta$) of the forces between slices (Figure C-12). The unknowns solved for and the equilibrium equations used are summarized in Table C-5.

The Modified Swedish Method is the name applied to force equilibrium procedures when they are used for analysis of circular slip surfaces. The Modified Swedish Method has been used extensively by the Corps of Engineers.

- Interslice forces have been represented in two ways in the Modified Swedish Method. In the first approach, the interslice forces are considered to represent the total forces between slices, the result of both effective stresses and pore water pressures. In the second approach, the side forces are considered to represent effective forces representing the effective stresses on the interslice boundaries. The forces resulting from water pressures are then considered as separate forces on the interslice boundaries. The computed value of the factor of safety will be different depending on the approach that is used.

- When total stresses are used to define the shear strengths in an analysis, e.g., for analyses with undrained strengths from UU (Q) tests, the interslice forces always represent total forces. In these cases, pore water pressures are not known, and thus, the forces from the water pressure on the sides of the slice cannot be calculated.

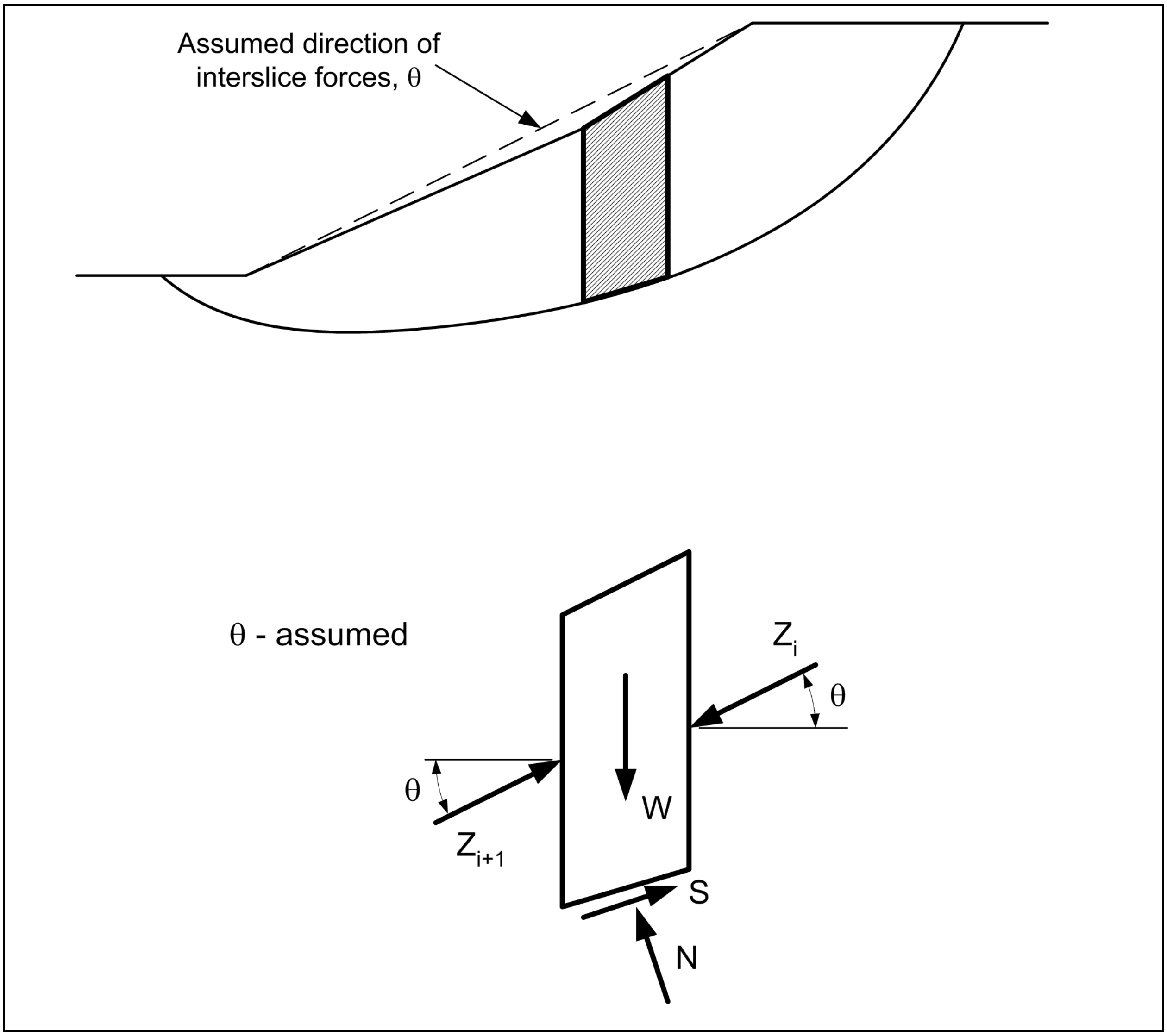



**Figure C-12. Typical slice and forces for Modified Swedish Method**

**Table C-5**
**Unknowns and Equations for Force Equilibrium Methods**

| **Unknowns** | **Number of Unknowns for n Slices** |
|---|---|
| Factor of safety (F) | 1 |
| Normal forces on bottom of slices (N) | n |
| Resultant interslice forces, Z | n – 1 |
| TOTAL NUMBER OF UNKNOWNS | 2n |
| **Equations** | **Number of Equations for n Slices** |
| Equilibrium of forces in the horizontal direction, $\Sigma F_x = 0$ | n |
| Equilibrium of forces in the vertical direction, $\Sigma F_y = 0$ | n |
| TOTAL NUMBER OF EQUILIBRIUM EQUATIONS | 2n |

- When effective stresses are used to define the shear strengths, e.g., for analyses of steady-state seepage, a choice can be made between having the interslice forces (Z) represent either the total force or only the effective force. If the interslice forces are chosen to represent the effective force, the corresponding forces due to water pressures on the sides of the slice are calculated and included as additional forces in the analysis. In the equations presented in this appendix, the interslice forces for the Modified Swedish Method are represented as effective forces when effective stresses are used to characterize the shear strength. However, the equations and examples with effective interslice forces can easily be converted to represent interslice forces as total forces by setting the forces that represent water pressures on the sides of the slice to zero.

- The original version of the Modified Swedish Method represented interslice forces as effective forces whenever effective stress analyses were performed (USACE 1970). In contrast, many computer programs represent the interslice forces as total forces. Fundamentally, representation of interslice forces as effective forces is sound and feasible for effective stress analyses because the pore water pressures are known (defined) when effective stress analyses are performed. However, there are a number of reasons why it is appropriate to represent interslice forces as total forces, particularly in computer software:

(1) In complex stratigraphy, it is difficult to define and compute the resultant force from water pressures on the sides of each slice.

(2) In many analyses, total stresses are used in some soil zones, and effective stresses are used in others; the shear strengths of freely draining soils are represented using effective stresses; while the shear strengths of less permeable soils are represented using undrained shear strengths and total stresses. Interslice water pressures can only be calculated when effective stresses are used for all materials. Thus, interslice forces must be represented as total forces in the cases where mixed drained and undrained shear strengths are used.

(3) It makes almost no difference whether interslice forces are represented as effective or total forces when complete static equilibrium is satisfied, e.g., when Spencer's Method is used to calculate the factor of safety. Thus, in Spencer's Method total interslice forces are almost always used. The Modified Swedish Method is recommended for hand-checking calculations made with Spencer's Method. Accordingly, when the Modified Swedish Method is used to check calculations made using Spencer's Method, it is logical that the interslice forces should be total forces.

- Regardless of whether the interslice forces represent total or effective forces, their inclination must be assumed. The inclination that is assumed is the inclination of either the total force or the effective force, depending on how the interslice forces are represented. The Corps of Engineers' 1970 manual states that the side forces should be assumed to be parallel to the "average embankment slope". The "average embankment slope" is usually taken to be the slope of a straight line drawn between the crest and toe of the slope (Figure C-12). All side forces are assumed to have the same inclination. The assumption of side forces parallel to the average embankment slope has been shown to sometimes lead to unconservative results in many cases – the calculated factor of safety is too large - when compared to more rigorous procedures which satisfy both force and moment equilibrium such as Spencer's Method or the Morgenstern and Price procedure. The degree of inaccuracy is greater when total interslice forces are used. It is probably more realistic and safer to assume that the interslice forces are inclined at one-half the average embankment slope when total forces are used.

- To avoid possibly overestimating the factor of safety, some engineers in practice have assumed that the interslice forces are horizontal in the Modified Swedish Method. The assumption of horizontal interslice forces in procedures that only satisfy force equilibrium, and not moment equilibrium, is sometimes referred to as the "Simplified Janbu" Method. This assumption, however, may significantly underestimate the value of the factor of safety. Accordingly, "correction" factors are sometimes applied to the value for the factor of safety calculated by the "Simplified Janbu" Method to account for the assumption of horizontal interslice forces (Janbu 1973). Some confusion exists in practice regarding whether the so-called "Simplified Janbu" Method should automatically include using the "correction" factors or not. Care should be exercised when reviewing results of slope stability calculations reported to have been made by the "Simplified Janbu" Method to determine whether a correction factor has been applied or not.

- Lowe and Karafiath (1960) suggested assuming that the interslice forces are inclined at an angle that is the average of the inclinations of the slope (ground surface) and shear surface at each vertical interslice boundary. Unlike the other assumptions described above, with Lowe and Karafiath's assumption the interslice force inclinations vary from slice to slice. This assumption appears to be better than any of the assumptions described earlier, especially when the side forces represent total, rather than effective, forces. Lowe and Karafiath's assumption produces factors of safety that are usually within 10 percent of the values calculated by procedures which satisfy complete static equilibrium (Duncan and Wright 1980).

(4) The force equilibrium equations for the Modified Swedish Method may be solved either graphically or numerically. Both the graphical and numerical solutions require an iterative, trial and error procedure to compute the factor of safety. A factor of safety is first assumed; force equilibrium is then checked. If force equilibrium is not satisfied, a new factor of safety is assumed and the process is repeated until force equilibrium is satisfied to an acceptable degree. The graphical and numerical procedures are each described separately in the sections that follow.

*b. Graphical solution procedure.* A solution for the factor of safety by any force equilibrium procedure (including the Modified Swedish Method) is obtained by repeatedly assuming a value for the factor of safety and then constructing the force vector polygon for each slice until force equilibrium is satisfied for all slices. A typical slice and the forces acting on it for a case where there is no surface or pore water pressure is shown in Figure C-12. The forces consist of the slice weight (W), the forces on the left and right sides of the slice ($Z_i$ and $Z_{i+1}$), and the normal and shear forces on the base of the slice (N and S). The interslice force, $Z_i$, represents the force on the upslope side of the slice, while $Z_{i+1}$ represents the force on the downslope side. Thus, $Z_i$ acts on the right side of the slice for a left facing slope and on the left side of the slice for a right-facing slope. The shear force on the bottom of the slice is expressed as:

$$S = \frac{1}{F}\left(c\Delta\ell + N\tan\phi\right) \qquad \text{(C-17)}$$

or

$$S = c_D\Delta\ell + N\tan\phi_D \qquad \text{(C-18)}$$

where $c_D$ and $\phi_D$ are the developed shear strength parameters.

In drawing the force polygons, the shear and normal forces are represented by a force resulting from cohesion, $c_D\Delta\ell$, and a force, $F_D$, representing the resultant force as a result of the normal force (N) and the frictional component of shear resistance ($N\tan\phi_D$). These forces are illustrated for a slice in Figure C-13b. The force $c_D\Delta\ell$ acts parallel to the base of the slice, while the force $F_D$ acts at an angle $\phi_D$ from the normal to the base

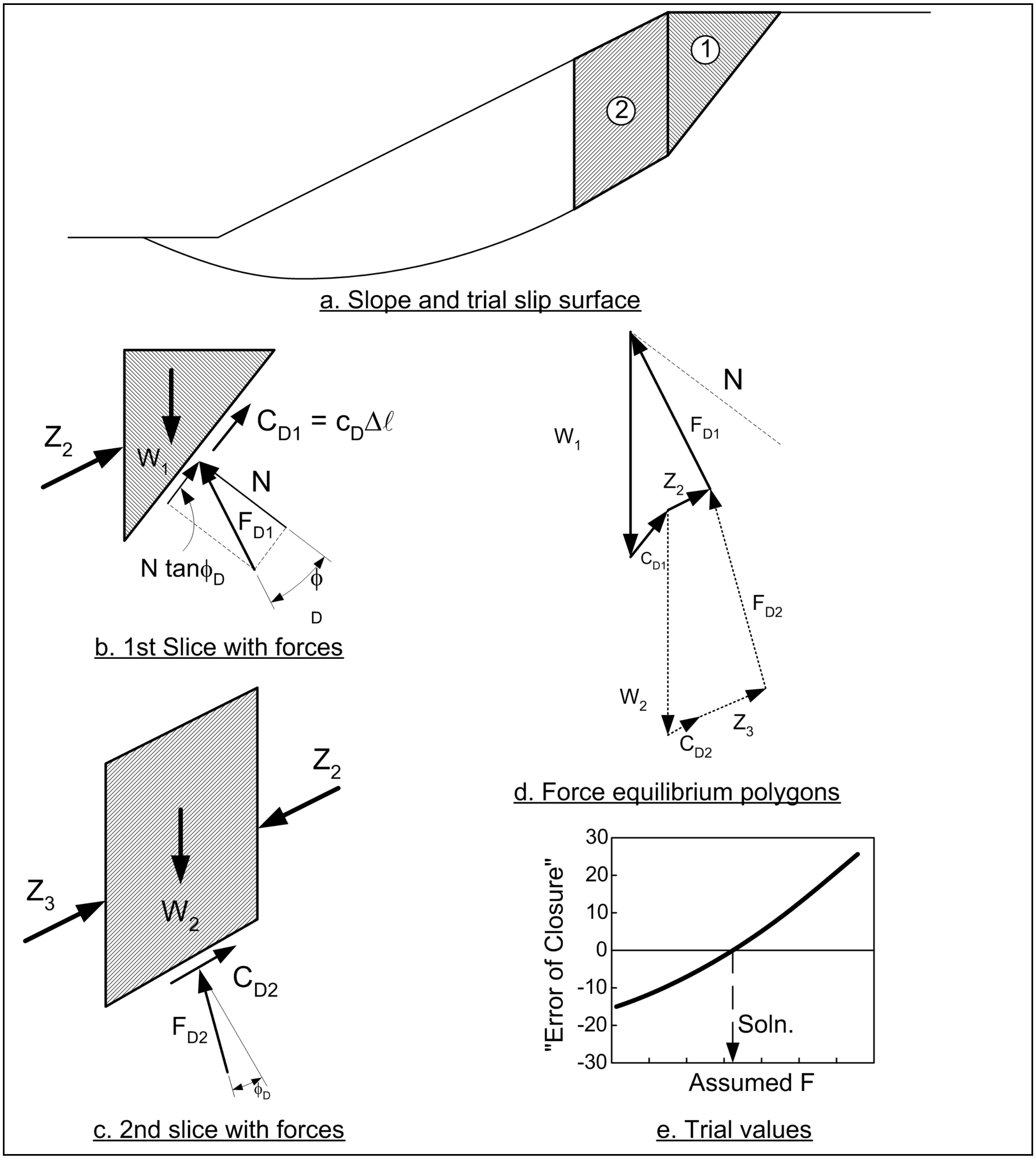



**Figure C-13. Forces and equilibrium force polygons for Modified Swedish Method**

of the slice. A value must be assumed for the factor of safety to construct the force polygons because $c_D$ and $\phi_D$ depend on the factor of safety. Once a factor of safety has been assumed and a suitable scale has been chosen, the force polygons are constructed for each slice as follows (Figure C-13d):

(1) A weight vector representing the weight (W) of the first slice is drawn vertically downward.

(2) A vector representing the force from the developed or developed or mobilized cohesion ( $c_D\Delta\ell$ ) is drawn in a direction parallel to the base of the slice, starting at the tip of the weight vector.

(3) A line representing the direction of the resultant force, $F_D$, is drawn so that the tip of the vector meets the start (tail) of the weight vector. The vector is drawn so that it makes an angle, $\phi_D$, with a line drawn perpendicular to the bottom of the slice and the shear component, $N\tan\phi_D$, is in the proper direction for the resisting force.

(4) A line representing the interslice force (Z) on the downslope side of the slice is drawn beginning at the end (tip) of the cohesion vector and extending in the direction assumed for the side forces. The intersection of this line with the line drawn in Step 3 defines the magnitude of the $F_D$ and Z vectors.

(5) The process is continued for the next slice, except the weight vector begins at the tip of the vector representing the cohesion force (Figure C-13d). The construction for slice 2 is shown by dotted lines.

(6) Vectors are drawn slice-by-slice until the last slice is reached. Because there is no force on the left side of the last slice, the force polygon should close with the resultant vector, $F_D$, alone. However, unless the correct value was assumed for the factor of safety, the force polygon will not close and an artificial force $Z_{i+1}$ is required to cause closure. This “error of closure” represents the force imbalance for the assumed factor of safety. Additional factors of safety must be assumed, and the error of closure is then plotted versus the trial factor of safety (Figure C-13e). Usually by plotting the results of three or four trials the factor of safety can be determined with acceptable accuracy. Further details of the equilibrium force polygons and solution are shown by the examples in Appendix F.

(7) A typical slice and the forces acting on it where the shear strength is expressed using effective stresses is shown in Figure C-14. The forces consist of the total weight of the slice (W), the water pressure forces on the left and right of the slice ($U_L$ and $U_R$), the side forces resulting from effective stresses ($Z_i$ and $Z_{i+1}$), the force resulting from developed or mobilized cohesion ($c'_D\Delta\ell$), the resultant force ($F'_D$) resulting from the effective normal force, N', and the frictional component of shear strength, $N'\tan\phi'$, and the force resulting from pore water pressures on the base of the slice ($U_b$). An additional force, P, will act on the top of the slice if the top of the slice is submerged. The forces W, $U_L$, $U_R$, $U_b$, and P are all known forces. To construct the force polygon these known forces are represented by a single resultant force R. The resultant force, R is represented graphically in Figure C-14c. The force will be vertical if there is no seepage (no flow); otherwise the force, R, will be inclined from vertical. Force polygons are constructed in a manner similar to that described above for no water pressures, except the vector, R, replaces the weight vector, W (Figure C-14d). Further details are shown by the examples in Appendix F.

*c. Numerical solution method.* In the numerical solution for any force equilibrium method (including the Modified Swedish Method), the side force on the downslope side of the slice is calculated using the following equation, derived from the equations of vertical and horizontal force equilibrium:

$$Z_{i+1} = Z_i + \frac{C_1 + C_2 + C_3 - C_4}{n_\alpha} \tag{C-19}$$

where

$$C_1 = W\left[\sin\alpha - \frac{\tan\phi'\cos\alpha}{F}\right] \tag{C-20a}$$

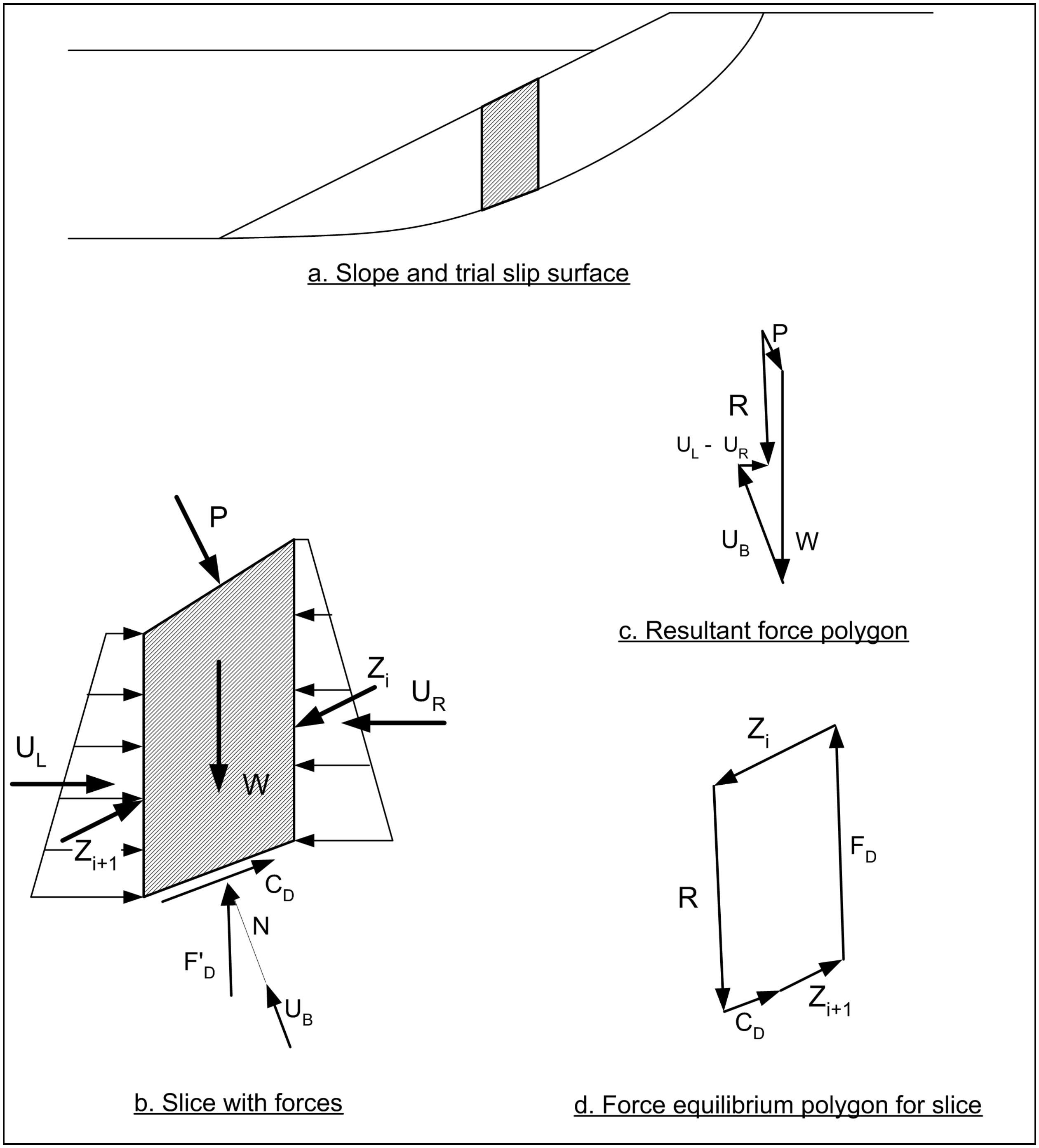



**Figure C-14. Forces for Modified Swedish Method with water**

$$C_2 = \left(U_i - U_{i+1}\right)\left[\cos\alpha + \frac{\tan\phi'\sin\alpha}{F}\right] \tag{C-20b}$$

$$C_3 = P\left[\sin\left(\alpha-\beta\right) - \frac{\tan\phi'}{F}\cos\left(\alpha-\beta\right)\right] \tag{C-20c}$$

$$C_4 = \left(c' - u \tan \phi'\right) \frac{\Delta \ell}{F} \tag{C-20d}$$

$$n_\alpha = \cos\left(\alpha - \theta\right) + \frac{\tan \phi' \sin\left(\alpha - \theta\right)}{F} \tag{C-21}$$

(1) The quantities $Z_i$ and $Z_{i+1}$ represent the forces on the upslope and downslope sides of the slice, respectively, $U_i$ and $U_{i+1}$ represent the water pressure forces on the upslope and downslope sides of the slice, and θ represents the inclination of the interslice forces. The remaining terms in Equation C-19 are the same as those defined earlier for the Ordinary Method of Slices and Simplified Bishop Methods. Equation C-19 is applied beginning with the first slice where $Z_i = 0$ and proceeding slice-by-slice until the last slice is reached. Here it is assumed that calculations are performed proceeding from the top of the slope to the bottom of the slope, regardless of the direction that the slope faces. The calculated interslice force $Z_{i+1}$ for the downslope side of the last slice (toe of the slip surface) should be zero if a correct value has been assumed for the factor of safety. If the force on the downslope side of the last slice is not zero, a new value is assumed for the factor of safety and the process is repeated until the force on the downslope side of the last slice is zero. Example calculations for the Modified Swedish Method using both the numerical solution and the graphical procedure are presented in Appendix F.

(2) When the quantities, $U_i$ and $U_{i+1}$, that represent water pressures on the sides of the slice are not zero, the interslice forces, $Z_i$ and $Z_{i+1}$, represent forces in terms of effective stress. When total stresses are used, the quantities, $U_i$ and $U_{i+1}$, are set to zero and the interslice forces then represent the total forces, including water pressures. The quantities, $U_i$ and $U_{i+1}$, can also be set equal to zero for effective stress analyses and the side forces are then the total side forces. Total interslice forces are used in much of the computer software for slope stability analyses, but effective forces are recommended when the side forces are assumed to be parallel to the average embankment slope, as discussed in Section C-4a.

*d. Limitations.* The principal limitation of the Modified Swedish Method is that calculated factors of safety are sensitive to the assumed interslice force inclination. Depending on the inclination assumed for the interslice forces, the factor of safety may be either underestimated or overestimated compared to the value calculated by more rigorous methods that fully satisfy static equilibrium. The sensitivity of the method appears to be due in large part to the fact that moment equilibrium is not satisfied.

*e. Recommendations for use.* The force equilibrium procedure is the only method considered to this point that can be utilized for analyses with general shaped, noncircular slip surfaces. Although the force equilibrium method is not as accurate as Spencer's Method (described next) for analyses of general-shaped noncircular slip surfaces, the force equilibrium method is much simpler and is therefore suitable for hand calculations, whereas Spencer's Method is too lengthy for hand calculations. Accordingly, the force equilibrium method is recommended for use in hand calculations where noncircular slip surfaces are being analyzed. If the force equilibrium method is being used to check calculations that were performed using Spencer's Method, the side force inclination used for the hand calculations should be the one calculated by Spencer's Method (Section C-5). Spencer's Method and the force equilibrium procedure should produce identical results when the same side force inclination is used in both method. The Modified Swedish Method is useful where existing slopes have been designed using the method and are being analyzed for new conditions, such as updated pore pressure information, or where alterations are to be made. Using the same method will allow meaningful comparison of results to those from previous analyses. For all new designs, preference should be given to the Simplified Bishop (circular slip surfaces) and Spencer (noncircular slip surfaces) Methods.

*f. Verification procedures.* As described above, either numerical or graphical procedures can be used in the Modified Swedish Method. Depending on which procedure was first used to compute the factor of safety (numerical or graphical), the other procedure can be used for verification. Thus, if the factor of safety was computed using the numerical procedure with Equation C-19, the force vector polygons can be drawn to confirm that force equilibrium has been satisfied. Likewise, if the graphical procedure was used to compute the factor of safety, the numerical solution (Equation C-19) can be used to compute the side forces and verify that equilibrium has been satisfied.

## C-5. Spencer's Method

*a. Assumptions.* Spencer's Method assumes that the side forces are parallel, i.e., all side forces are inclined at the same angle. However, unlike the Modified Swedish Method, the side force inclination is not assumed, but instead is calculated as part of the equilibrium solution. Spencer's Method also assumes that the normal forces on the bottom of the slice act at the center of the base – an assumption which has very little influence on the final solution. Spencer's Method fully satisfies the requirements for both force and moment equilibrium. The unknowns and equations involved in the method are listed in Table C-6.

**Table C-6**
**Unknowns and Equations for Spencer's Methods**

| **Unknowns** | **Number of Unknowns for n Slices** |
|---|---|
| Factor of safety (F) | 1 |
| Inclination of interslice forces (θ) | 1 |
| Normal forces on bottom of slices (N) | n |
| Resultant interslice forces, Z | n – 1 |
| Location of interslice normal forces | n – 1 |
| TOTAL NUMBER OF UNKNOWNS | 3n |
| **Equilibrium Equations** | |
| **Equations** | **Number of Equations for n Slices** |
| Equilibrium of forces in the horizontal direction, $\Sigma F_x = 0$ | n |
| Equilibrium of forces in the vertical direction, $\Sigma F_y = 0$ | n |
| Equilibrium of moments | n |
| TOTAL NUMBER OF EQUILIBRIUM EQUATIONS | 3n |

Although Spencer (1967) originally presented his method for circular slip surfaces, Wright (1969) showed that the method could readily be extended to analyses with noncircular slip surfaces. A solution by Spencer's Method first involves an iterative, trial and error procedure in which values for the factor of safety (F) and side force inclination (θ) are assumed repeatedly until all conditions of force and moment equilibrium are satisfied for each slice. Then the values of N, Z, and $y_t$ are evaluated for each slice.

*b. Limitations.* Spencer's Method requires computer software to perform the calculations. Because moment and force equilibrium must be satisfied for every slice and the calculations are repeated for a number of assumed trial factors of safety and interslice force inclinations, complete and independent hand-checking of a solution using Spencer's Method is impractical (Section C-5d).

*c. Recommendations for use.* The use of Spencer's Method for routine analysis and design has become practical as computer resources improve. The method has been implemented in several commercial computer programs and is used by several government agencies. Spencer's Method should be used where a statically complete solution is desired. It should also be used as a check on final designs where the slope stability computations were performed by simpler methods.

*d. Verification procedures.* Complete and independent hand-checking of a solution using Spencer's Methods is impractical because of the complexity of the method and the lengthy calculations involved. Instead the force equilibrium procedure is recommended, using either the graphical or numerical solution methods. When checking Spencer's Method using the force equilibrium procedure, the side force inclination

($\theta$) is assumed to be the same as the one found using Spencer's Method. In this case (same side force inclination), both the force equilibrium procedure and Spencer's Method should produce the same value for the factor of safety.

## C-6. The Wedge Method

*a. Assumptions.* The Wedge Method is illustrated in Figure C-15. The method assumes that the sliding mass is composed of three regions: the active wedge, the central block, and the passive wedge. The inclination of the forces on the vertical boundaries between the zones are assumed. The Wedge Method is actually a special case of the force equilibrium procedure: the Wedge Method fully satisfies equilibrium of forces in the vertical and horizontal directions and ignores moment equilibrium. The only differences between the Wedge Method and the Modified Swedish Method are (1) the assumptions for the shape of the potential sliding surface, and (2) possibly, the inclinations of the "interslice" forces between wedges. In the Wedge Method, the interslice force inclination assumption is often made the same as for the Modified Swedish Method. However, the interslice force between the central block and the passive wedge is sometimes assumed to be horizontal.

*b. Solution procedure.* Solutions for the Wedge Method are the same as for any of the force equilibrium procedures (Section C-4).

*c. Limitations.* The Wedge Method has the same limitations as other force equilibrium procedures. In addition, the specific, "wedge" shape of the slip surface restricts use of the procedure to slopes where slip surfaces of this shape are likely to be critical.

*d. Recommendations for use.* Factors of safety calculated using the Wedge Method are sensitive to the assumed inclinations of the side forces. The Wedge Method may be used to check Spencer's solutions for three-part noncircular shear surfaces. The side force inclination is taken as the same side force inclination found in Spencer's. The Wedge Method also has use where existing slopes have been designed using the method and are being analyzed for new conditions, such as updated pore pressure information, or where alterations are to be made. Using the same method allows meaningful comparison of results to those from previous analyses. For all new designs, preference should be given to complete analysis procedures such as Spencer's Method, which can be used for noncircular and wedge-shaped shear surfaces.

*e. Verification procedures.* The same procedures, graphical or numerical, used to verify calculations performed by the Modified Swedish Method, may be used to verify calculations by the Wedge Method.

## C-7. The Infinite Slope Method

*a. Assumptions.* The Infinite Slope Method assumes that the slope is of infinite lateral extent and that sliding occurs along a plane surface parallel to the face of the slope (Figure C-16). For slopes composed of uniform cohesionless soils ($c' = 0$), the critical slip surface will be parallel to the outer slope at small depth ($z \approx 0$). In this situation, the instability mechanism involves individual soil particles rolling down the face of the slope. Analyses of this condition using circular slip surfaces will result in a critical circle that approximates the infinite slope failure mechanism with a circle that is very shallow and has a very large radius. The factor of safety will be the same as calculated using an infinite slope analysis. However, the infinite slope analysis is simpler and easier, and it should be used for slopes in cohesionless materials. The Infinite Slope Method is a special case of the force equilibrium procedure, with one slice. With only one slice, two equations are available (horizontal and vertical force equilibrium), and two unknowns must be evaluated (the factor of safety and the normal force on the base of the slice). Thus, the method is statically determinate.

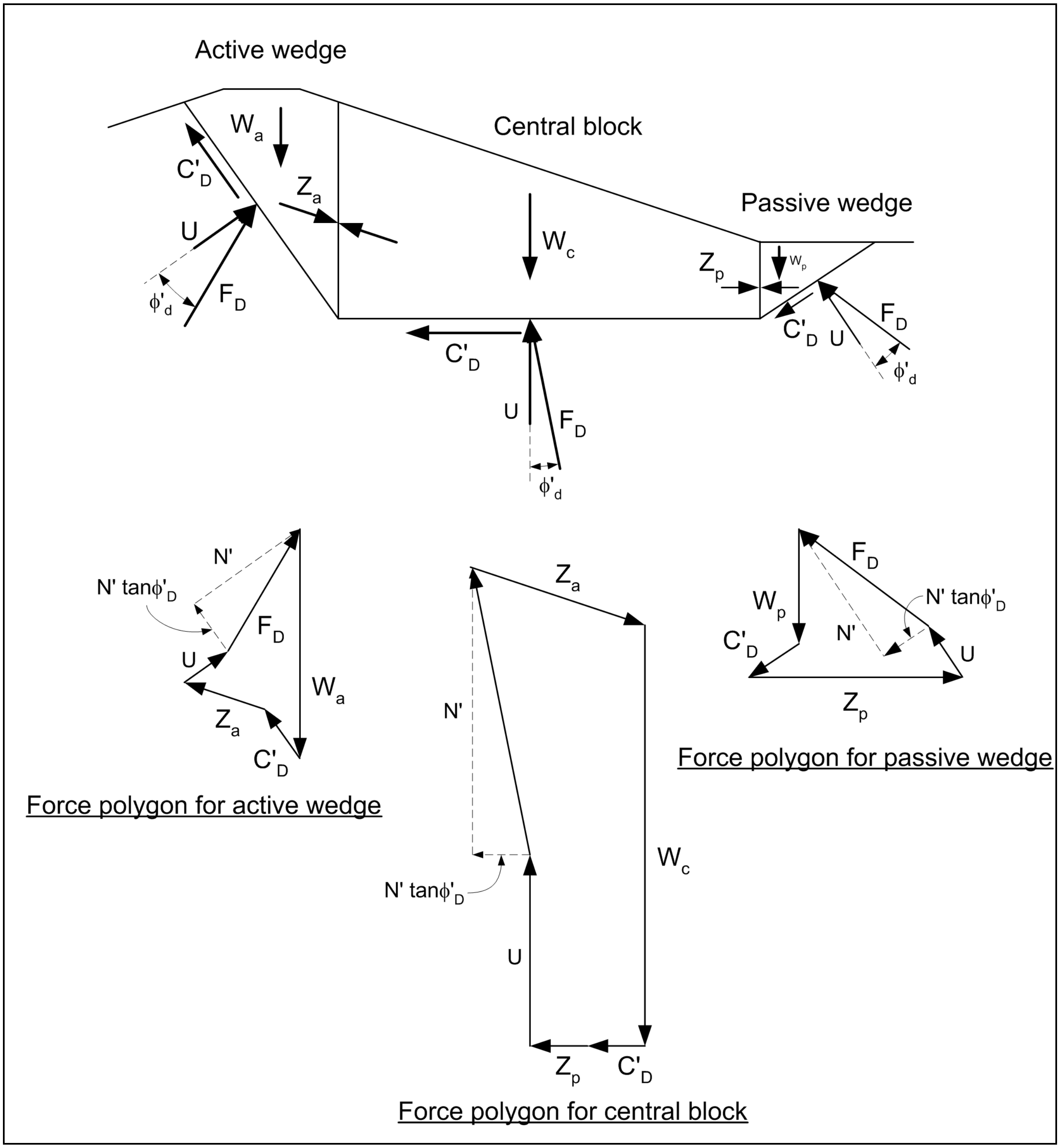



**Figure C-15. Forces and equilibrium polygons for Wedge Method**

*b. Stability equations.* For an infinite slope, the total normal and shear stresses on a plane parallel to the slope at a vertical depth, z, are given by:

$$\sigma = \gamma z \cos^2 \beta \tag{C-22}$$

and

$$\tau = \gamma z \cos\beta \sin\beta \tag{C-23}$$

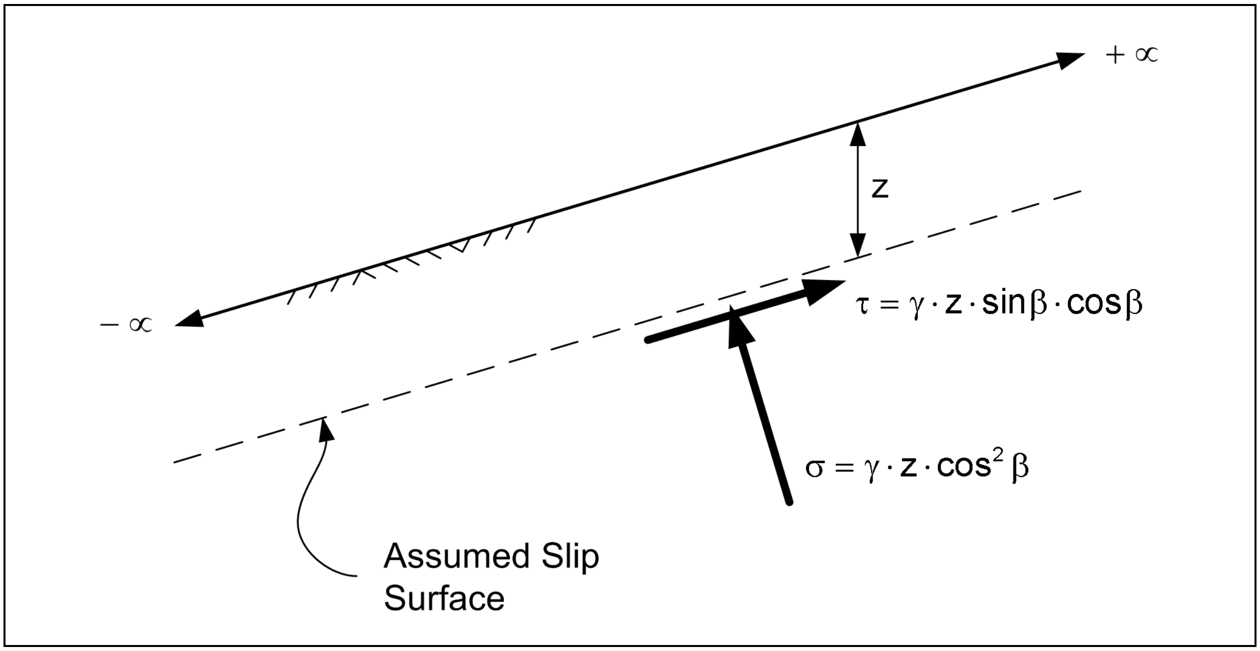



**Figure C-16. Infinite slope**

For shear strengths expressed in terms of effective stresses and zero cohesion (c' = 0), the factor of safety is given by:

$$F = \frac{s}{\tau} = \frac{(\sigma - u)\tan\phi'}{\tau} \tag{C-24}$$

where u is the pore water pressure at the depth of the shear plane.

Letting $r_u = u/\gamma z$ and substituting the expressions for $\sigma$ and $\tau$ from Equations C-22 and C-23 into Equation C-24, gives:

$$F = \frac{s}{\tau} = \frac{(\cos^2\beta - r_u)\tan\phi'}{\cos\beta\sin\beta} \tag{C-25}$$

Equation C-25 can also be written as:

$$F = \frac{\tan\phi'}{\tan\beta}\left[1 - r_u\left(1 + \tan^2\beta\right)\right] \tag{C-26}$$

For the special case of no pore water pressure (u = 0; $r_u$ = 0) Equation C-26 reduces to:

$$F = \frac{\tan\phi'}{\tan\beta} \tag{C-27}$$

(1) The stability equation for an infinite slope can also be written for conditions involving seepage through the slope, as shown in Figure C-17.

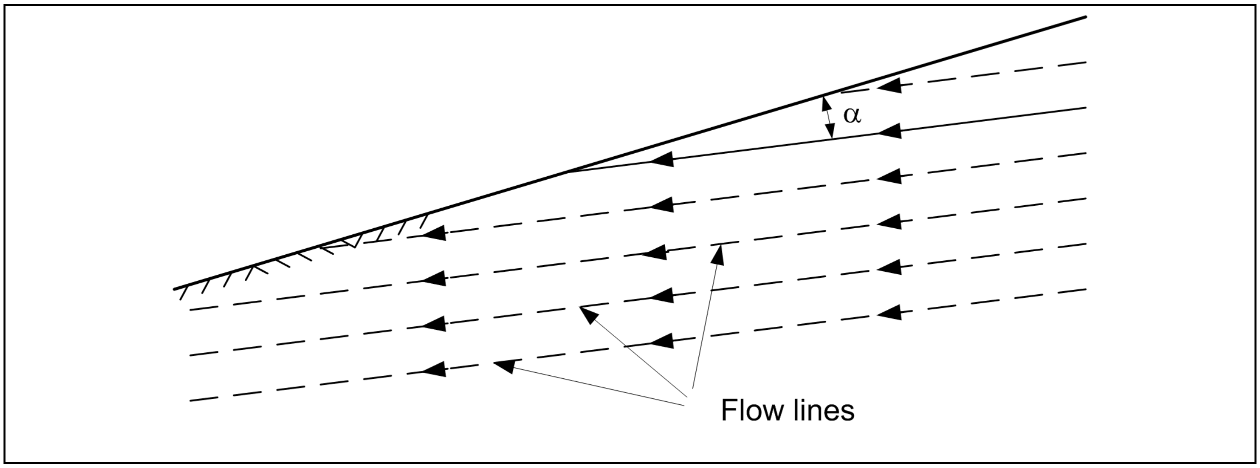



**Figure C-17. Infinite slope with parallel flow lines**

The factor of safety for an infinite slope with seepage can be expressed as follows (Bolton 1979):

$$F = \frac{\gamma' - \gamma_w \tan\alpha_s \tan\beta}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta} \tag{C-28}$$

where

$\gamma' = \gamma_{sat} - \gamma_w$ = submerged unit weight of soil

$\gamma_w$ = unit weight of water

$\gamma_{sat}$ = saturated unit weight of soil

$\alpha_s$ = angle between the flow lines and the embankment face (Figure C-17)

$\beta$ = inclination of the slope measured from the horizontal

$\phi'$ = angle of internal friction expressed in terms of effective stresses

The cohesion, c', is assumed to be zero because the infinite slope analysis is primarily applicable to cohesionless soils.

(2) For the case where the direction of seepage is parallel to the slope ($\alpha_s = 0$), with the free surface of seepage at the ground surface, the factor of safety can be expressed as:

$$F = \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta} \tag{C-29}$$

Similarly, for the case of horizontal seepage ($\alpha = \beta$)

$$F = \frac{\gamma' - \gamma_w \tan^2\beta}{\gamma_{sat}} \frac{\tan\phi'}{\tan\beta} \tag{C-30}$$

*c. Limitations.* The equations for infinite slope factor of safety given by Equations C-24 through C-30 are applicable only to slopes in cohesionless materials. They apply to slopes in nonplastic silt, sand, gravel, and rock-fill where c' = 0. Charts for analysis of infinite slopes in materials with c' > 0 are given in Appendix E.

*d. Recommendations for use.* The method is useful for evaluating the stability with respect to shallow sliding of slopes in cohesionless soils.

## C-8. Simple Approximations

Simple approximations are sometimes useful for preliminary estimates of stability prior to more rigorous and complete calculations. Two such simplified approaches are discussed below.

*a. At-rest earth pressure method.* The at-rest earth pressure method is used to estimate the potential for lateral spreading and horizontal sliding of an embankment, as shown in Figure C-18.

(1) Assumptions. The method compares the at-rest earth pressure on a vertical plane through the embankment to the shear resistance along the base of the embankment to one side of the plane. The method is only partly a limit-equilibrium method, because the at-rest earth pressures are calculated independently of any equilibrium conditions and, then, compared to the limiting shear resistance.

(2) Limitations. The method is not intended as a primary design method but only as a method to perform supplemental checks. It is applicable only to embankments.

(3) Recommendations for use. Ensuring that an embankment has an adequate factor of safety by this analysis will assist in limiting deformations where two or more materials with significantly different stress-strain behavior are present. A common example application is to zoned gravel or rock-fill dams with clay cores.

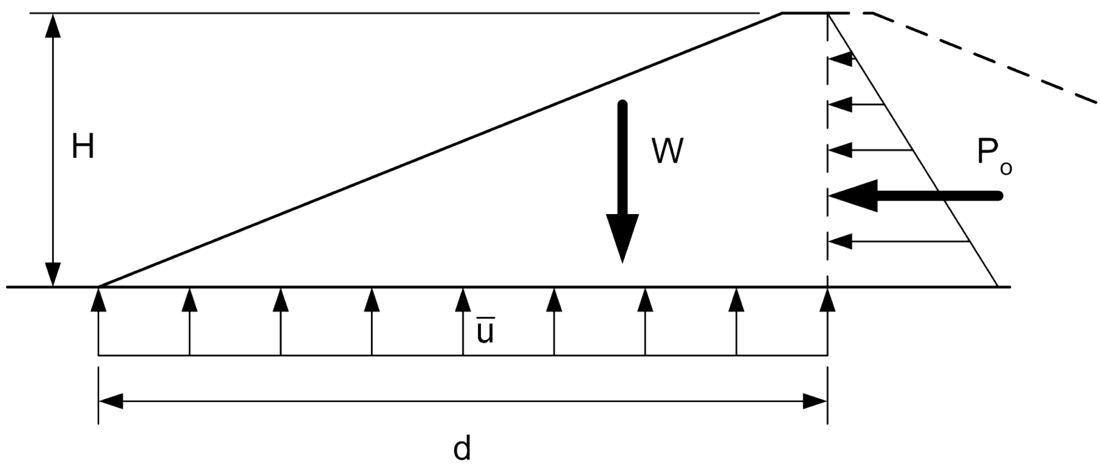
*b. Bearing capacity methods.* Bearing capacity methods are useful for estimating the potential for weak, saturated, clay foundations to support embankments (Figure C-19).

(1) Assumptions. These methods compare the ultimate bearing capacity of the foundation beneath an embankment to the total vertical stress imposed by the embankment. The vertical stress is calculated by multiplying the full height of the embankment by the total unit weight of the fill material. The bearing capacity of the foundation is calculated from the classical bearing capacity equations for a strip footing resting on the surface of the ground. For a saturated clay and undrained loading ($\phi = 0$), the ultimate bearing capacity is computed as:

$$q_{ult} = 5.14c \qquad \text{(C-31)}$$

Although more sophisticated approximations can be made, bearing capacity analyses should not be considered to be a substitute for detailed slope stability analyses.

(2) Limitations. The bearing capacity methods are limited to homogeneous foundations where simple bearing capacity equations are applicable. These methods are also used primarily for evaluating short-term, undrained stability of embankments resting on soft, saturated clay foundations. These methods are intended only for preliminary analyses and for use as an approximate check of more rigorous and thorough analyses.




$$F = \frac{c' \cdot d + (W - \bar{u}d) \cdot \tan\phi'}{P_0}$$

$P_o$ = total force due to at-rest pressures on vertical plane through center of embankment

c' and $\phi'$ = shear strength parameters along horizontal surface between embankment and foundation

$\bar{u}$ = average pore water pressure along embankment-foundation contact

**Figure C-18. At-rest earth pressure method**

(3) Recommendations for use. This simple bearing capacity approach ignores the shear strength of the embankment fill and is conservative in this respect. Because the shear strength of the embankment material is ignored, questions about incompatibility between the stress-strain behavior of the embankment and the foundation do not arise.

## C-9. Chart Solutions

a. Chart solutions are very useful for obtaining preliminary estimates of stability and for checking detailed analyses. For cases where the conditions represented by the stability charts match those of the actual slope, charts provide an accurate value of factor of safety. In such cases, factors of safety computed using charts are more accurate than those computed using procedures such as the Ordinary Method of Slices and the Modified Swedish Method.

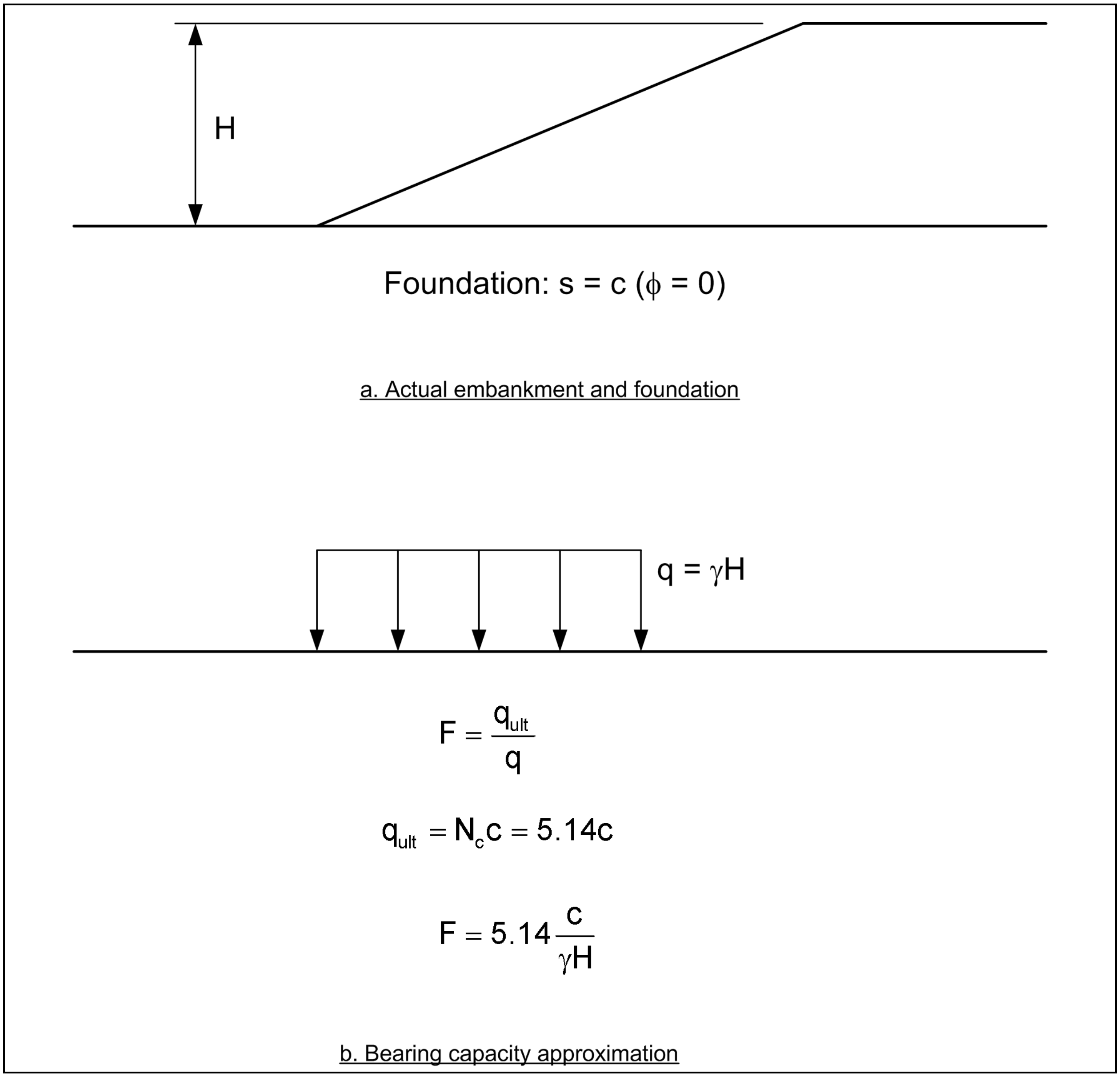



**Figure C-19. Use of bearing capacity methods to estimate stability of embankments on soft, saturated clay foundations**

*b.* In addition to charts derived from analyses using limit equilibrium procedures, like those described in the previous sections, charts based on field observations of slope performance have also been developed. This second type of chart includes effects of geologic and groundwater conditions, which is advantageous, but such charts are only useful for the area and the types of slopes for which they are developed.

*c.* Charts developed using analytical methods are discussed in detail in Appendix E.

## C-10. Acceptability of Solutions and Computational Problems

*a. General.* The assumptions introduced to render slope stability problems statically determinate sometimes lead to unrealistic solutions. Regardless of the method used, calculated results must be checked to identify computational problems. Calculated values of normal forces (N) and interslice forces (Z) should be examined to be sure that their values are reasonable. Because most soils are not able to sustain significant tensile stresses, tensile forces should not exist on the sides or bottom of slices. Also the line of thrust (the locus of points describing the location of the interslice forces) should be within the sliding mass. Several specific computational problems are discussed below.

*b. Very large forces or tensile forces due to slip surface geometry.* As shown in Figure C-20, the resultant force on the slip surface ($F_D$) can become parallel or nearly parallel to the interslice force (Z) if the slip surface exits too steeply at the toe. When this condition occurs, very large, infinite, or negative, values may be calculated for these forces (Ching and Fredlund 1983). If $F_D$ and Z are parallel, division by zero occurs in the equilibrium equations, and the forces become infinite. If $F_D$ and Z are close to parallel, division by a very small number occurs in the equilibrium equations, and the values of $F_D$ and Z can be very large, either positive or negative. Factors of safety computed for such conditions are not meaningful. The condition of large positive or negative forces near the toe of the slope is usually caused by the slip surface exiting upward too steeply. The problem can be avoided by adjusting the inclination of the slip surface to conform more closely with the most critical slip surface that would be expected based on passive earth pressure theories. In the case where the ground surface and earth pressure (interslice) force are both horizontal, the inclination of the critical slip surface (shear plane) for passive earth pressure conditions is given by:

$$\alpha = -\left(45^\circ - \frac{\phi'}{2}\right) \tag{C-32}$$

The negative sign arises from the sign convention used for the inclination of the shear surface in the slope stability equations. In the case of an inclined earth pressure (interslice) force, the inclination of the critical slip surface can be calculated from the following equation presented by Jumikis (1962):

$$\alpha = -\Omega + \phi \tag{C-33}$$

where

$$\Omega = \arctan\left[\frac{\tan\phi + \sqrt{\tan\phi(\tan\phi + \cot\phi)(1 + \tan\delta\cot\phi)}}{1 + \tan\delta(\tan\phi + \cot\phi)}\right] \tag{C-34}$$

where $\delta$ is the inclination of the earth pressure force, which corresponds to $\theta$ in Figure C-20.

The sign convention for $\alpha$ in Equation C-33 is such that $\alpha$ is negative for slip surfaces inclined upward at the toe of the slope. The existence of very large positive or negative values for the forces near the toe of the slope can lead to unreasonably large or unreasonably small values for the factor of safety. Depending on the procedure of slope stability analysis being used, the problem can be avoided in one of the following ways:

(1) The slip surface can be flattened near the toe as described above: This is probably the best approach, but the use of noncircular slip surfaces is required.

(2) The side force inclination can be changed: Of the procedures described in this manual, the Modified Swedish Method is the only one that allows the inclination of the side forces to be changed. It is also possible to change the assumed inclination for the side force with using the Morgenstern and Price method (Morgenstern and Price 1965). Changing the side force inclination to obtain a suitable solution with the Morgenstern and Price procedure can be time-consuming.

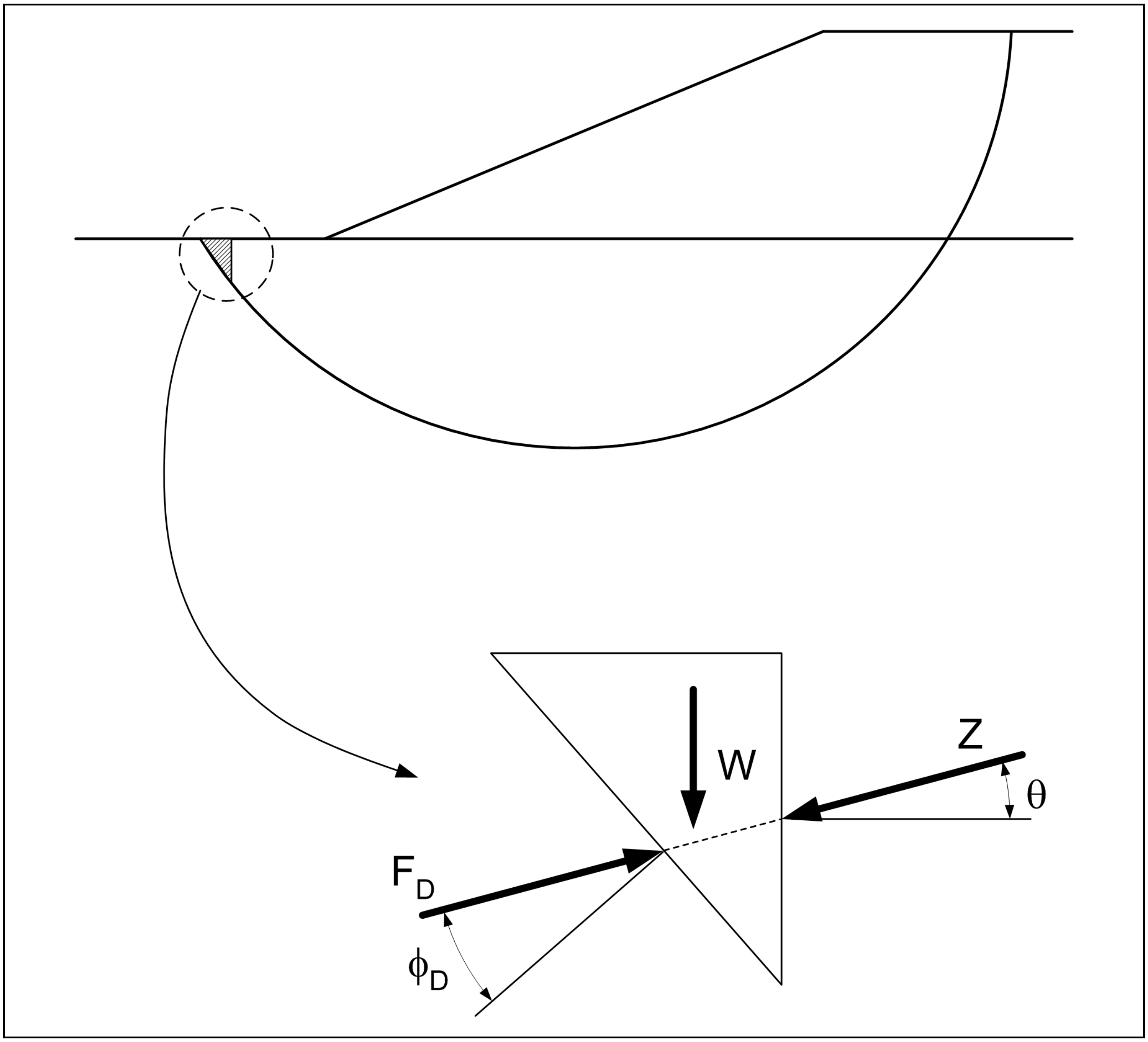



**Figure C-20. Slice with parallel (co-linear) resultant force, $F_D$, and interslice force, $Z$, leading to infinite values of these forces**

(3) The Ordinary Method of Slices can be used for the analysis. The problem described above does not occur with the OMS, because the OMS neglects side forces. However, the OMS is not accurate for effective stress analyses when pore pressures are high, and its use is undesirable for that reason.

(4) The shear strength in the zone where the slip surface exits can be estimated assuming a simple passive earth pressure state of stress. The shear strength is then assigned to this zone as a cohesion with $\phi = 0$. For cohesionless soil ($c = 0$), horizontal ground surface, and a horizontal earth pressure force, the shear strength $s_{passive}$ can be calculated from:

$$s_{passive} = \frac{1}{2}\left[\tan^2\left(45° + \frac{\phi'}{2}\right) - 1\right]\sigma'_v \cos\phi' \tag{C-35}$$

where $\sigma'_v$ is the effective vertical stress.

In this case, the shear strength increases linearly with depth because the effective vertical stress also increases linearly with depth. Approach (4) is the only one that can be used to eliminate large positive or negative forces at the toe of the slope when the Simplified Bishop Method is used. Regardless of the procedure used to calculate the factor of safety, the details of the solution should be examined to determine if very large positive or negative forces are calculated for slices near the toe of the slope. If such conditions are found, one of the measures described above should be used to correct the problem.

*c. Tensile forces from cohesion*. When soils at the crest of the slope have cohesion, the calculated values for the normal forces (N) and side forces (Z) in this area are often negative. Negative forces are consistent with what would be calculated by classical earth pressure theories for the active condition. The negative stresses result from the tensile strength that is implicit for any soil having a Mohr-Coulomb failure envelope with a cohesion intercept. This type of shear strength envelope implies that the soil has tensile strength, as shown in Figure C-21. Because few soils have tensile strength that can be relied on for slope stability, tensile stresses should be eliminated before an analysis is considered acceptable. Tensile stresses can be eliminated from an analysis by introducing a vertical tension crack near the upper end of the slip surface. The slip surface is terminated at the point where it reaches the bottom of crack elevation, as shown in Figure C-22. The appropriate crack depth can be determined in either of the following ways:

(1) A range of crack depths can be assumed and the factor of safety calculated for each depth. The crack depth producing the minimum factor of safety is used for final analyses. The depth yielding the minimum factor of safety will correspond closely to the depth where tensile stresses are eliminated, but positive (driving) stresses are not.

(2) The crack depth can be estimated as the depth over which the active Rankine earth pressures are negative. For total stresses and homogeneous soil the depth is given by:

$$d_{crack} = \frac{2c_D}{\gamma \tan\left(45° - \frac{\phi_D}{2}\right)} \tag{C-36}$$

where

$c_D$ and $\phi_D$ = developed shear strength parameters

$c_D$ = c/F

$\tan \phi_D$ = $\tan \phi$/F

Similar expressions can be developed for the depth of tension for effective stresses and/or nonhomogeneous soil profiles. In some cases the depth of crack computed using Equation C-36 will be greater than the height of the slope. This is likely to be the case for low embankments of well-compacted clay. For embankments on weak foundations, where the crack depth computed using Equation C-36 is greater than the height of the embankment, the crack depth used in the stability analyses should be equal to the height of the embankment; the crack should not extend into the weak foundation.

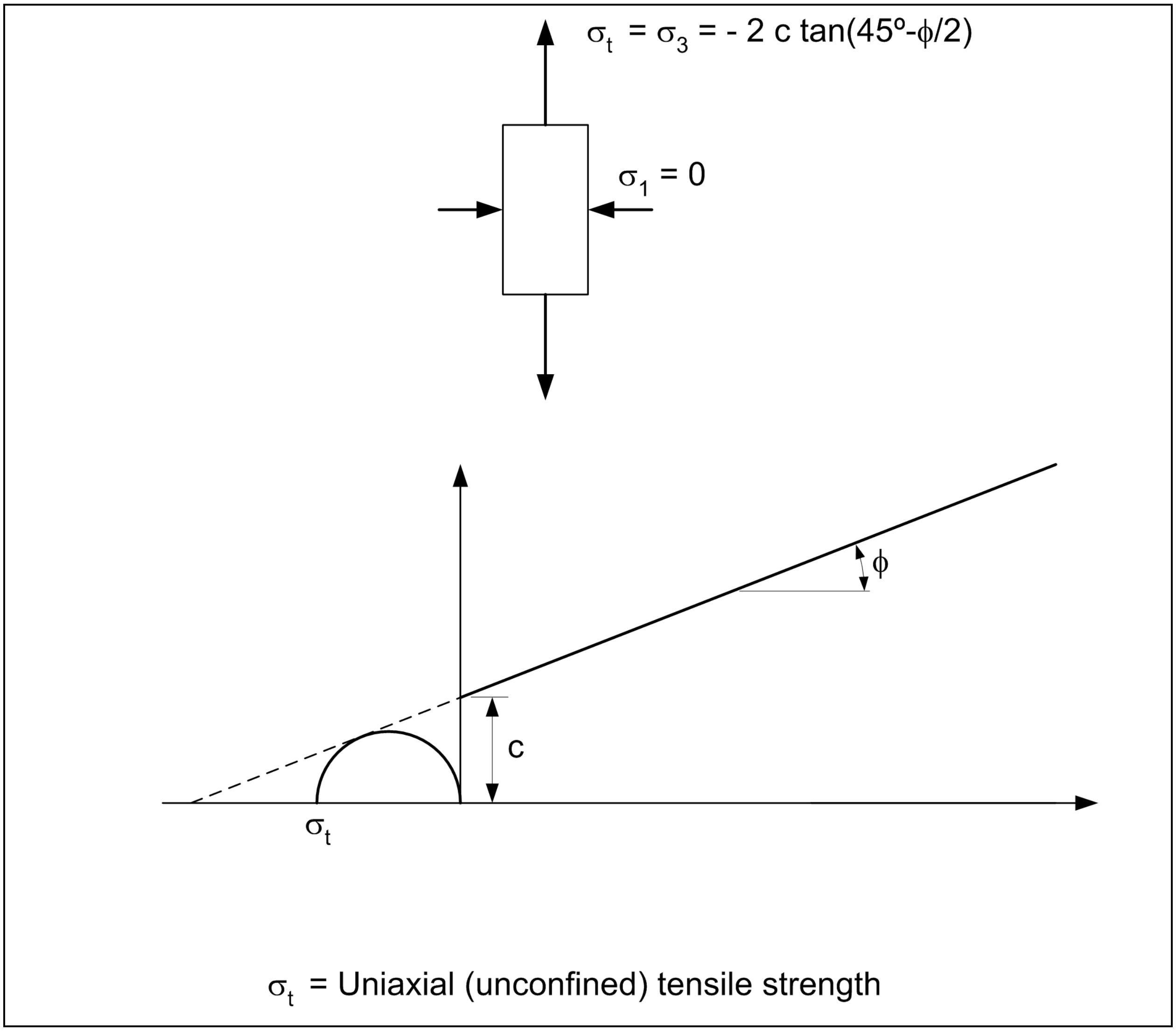



**Figure C-21. Tensile strength implied by a Mohr-Coulomb failure envelope with a cohesion intercept**

*d. Nonconvergence.* The Simplified Bishop, Modified Swedish, and Spencer's Methods all require iterative procedures to calculate the factor of safety. In certain cases, the trial and error solution may not converge. In very rare cases, the same data and slip surface can yield two different solutions for the factor of safety, depending on the initial value of factor of safety used in the iterative procedure. These difficulties can be avoided using one or more of the following measures:

(1) Use reasonable slip surface inclinations at the bottom of the slip surface, and use tension cracks at the top to eliminate tensile stresses. In essentially all cases where multiple solutions for the factor of safety are found for a given slip surface and data, one of the solutions is clearly unrealistic because of unusually large or negative stresses near the toe of the slope. Such inappropriate solutions can be easily identified and rejected.

(2) Avoid unrealistic initial estimates for the factor of safety. Iterative schemes implemented in computer programs usually limit the number of iterations and the amount by which the factor of safety can change from one iteration to the next. If the initial estimate of the factor of safety is far from the correct value, the solution may not be reached within the allowable number of iterations. In most cases this is not a problem because factors of safety will range from 0.5 to 3, and an estimate in this range is usually close enough.

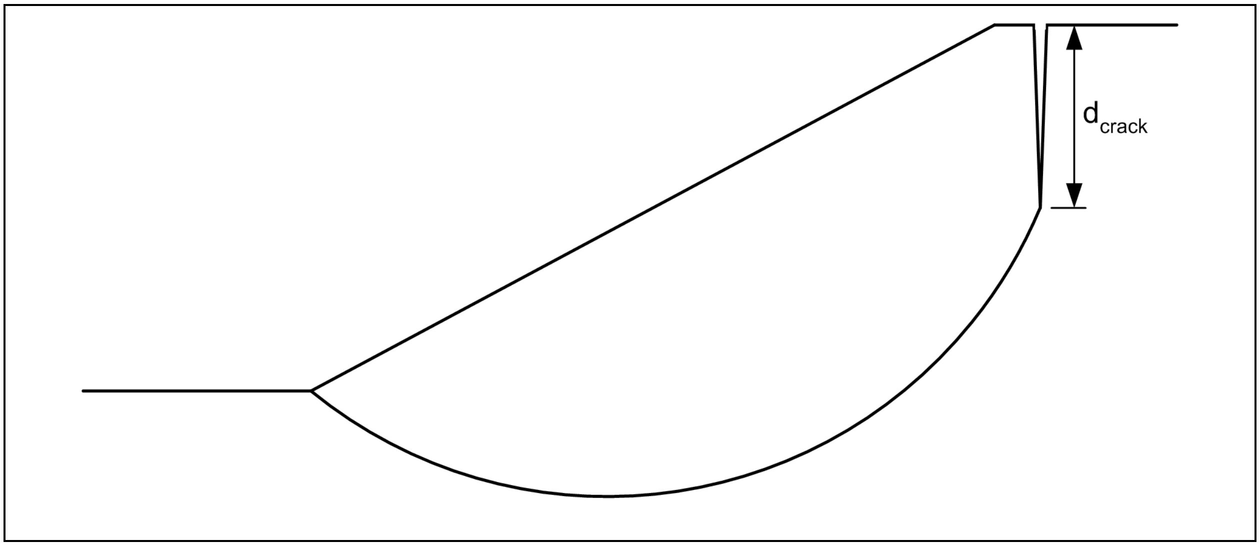



**Figure C-22. Vertical crack introduced to eliminate tension near the crest of a slope**

(3) It is often better to overestimate the initial value for the factor of safety, rather than underestimate the value. Experience with Spencer's Method in particular has revealed that the solution generally converges best when the initial trial value for the factor of safety is greater, rather than less, than the final value.

(4) Avoid unrealistic problems. For example, it is possible to prescribe either external loads or internal reinforcement forces that are large enough to make the slope stable with no shear strength mobilized. In fact, it is even possible to specify forces that are sufficiently large to cause the soil mass to fail in an upslope direction. In such cases, solutions usually fail to converge. To obtain a solution in these cases of upslope failure, either the factor of safety has to be treated as a negative quantity or the direction assumed for the shear force (S) has to be reversed. Most software cannot automatically recognize this, and the solution will not converge.

(5) Use realistic estimates for the position of the initial trial slip surface in an automatic search. If the initial estimate for the slip surface is not realistic, the computer software may be unable to compute the factor of safety and the automatic search may not be able to continue. Alternatively, the search may continue, but never reach a reasonable slip surface.

*e. Other numerical problems.* All computer software for slope stability computations requires extensive numerical computations related to the slope and soil profile geometry, and roundoff and truncation errors can occur in these calculations. Computed results should be examined to check for the possibility of such errors. The following measures will reduce problems with roundoff and truncation errors:

(1) Avoid placing the origin of the coordinate system very far from the slope, such that coordinates are very large with relatively little difference between them, e.g., 1000001 vs. 1000002.

(2) Avoid very nearly vertical, but not vertical boundaries between materials, slope faces, etc. Some computer programs do not allow vertical boundaries and/or slopes and require use of slightly inclined boundaries. Problems may occur if a boundary is only very slightly inclined.

(3) If possible, avoid potential slip surfaces that cross material boundaries at extremely flat angles. This may cause numerical problems in calculating intersections. The numerical differences in the slopes of two lines frequently appear in the denominator of an expression, and if the difference in slopes is small, but not

zero, this may cause errors. As a result of roundoff and computer word length, the calculated point of intersection can be a considerable distance from the actual point of intersection.

## C-11. Selection of Method

Some methods of slope stability analysis (e.g., Spencer's) are more rigorous and should be favored for detailed evaluation of final designs. Some methods (e.g., Spencer's, Modified Swedish, and the Wedge) can be used to analyze noncircular slip surfaces. Some methods (e.g., the Ordinary Method of Slices, the Simplified Bishop, the Modified Swedish, and the Wedge) can be used without the aid of a computer and are therefore convenient for independently checking results obtained using computer programs. Also, when these latter methods are implemented in software, they execute extremely fast and are useful where very large numbers of trial slip surfaces are to be analyzed. The various methods covered in this appendix are summarized in Table C-7. This table can be helpful in selecting a suitable method for analysis.

**Table C-7**
**Comparison of Features of Limit Equilibrium Methods**

| Feature | Ordinary Method of Slices | Simplified Bishop | Spencer | Modified Swedish | Wedge | Infinite Slope |
|---|---|---|---|---|---|---|
| Accuracy | | X | X | | | X |
| Plane slip surfaces parallel to slope face | | | | | | X |
| Circular slip surfaces | X | X | X | X | | |
| Wedge failure mechanism | | | X | X | X | |
| Non-circular slip surfaces – any shape | | | X | X | | |
| Suitable for hand calculations | X | X | | X | X | X |

## C-12. Use of the Finite Element Method

*a. General.* The finite element method (FEM) can be used to compute displacements and stresses caused by applied loads. However, it does not provide a value for the overall factor of safety without additional processing of the computed stresses. The principal uses of the finite element method for design are as follows:

(1) Finite element analyses can provide estimates of displacements and construction pore water pressures. These may be useful for field control of construction, or when there is concern for damage to adjacent structures. If the displacements and pore water pressures measured in the field differ greatly from those computed, the reason for the difference should be investigated.

(2) Finite element analyses provide displacement pattern which may show potential and possibly complex failure mechanisms. The validity of the factor of safety obtained from limit equilibrium analyses depends on locating the most critical potential slip surfaces. In complex conditions, it is often difficult to anticipate failure modes, particularly if reinforcement or structural members such as geotextiles, concrete retaining walls, or sheet piles are included. Once a potential failure mechanism is recognized, the factor of safety against a shear failure developing by that mode can be computed using conventional limit equilibrium procedures.

(3) Finite element analyses provide estimates of mobilized stresses and forces. The finite element method may be particularly useful in judging what strengths should be used when materials have very dissimilar stress-strain and strength properties, i.e., where strain compatibility is an issue. The FEM can help

identify local regions where "overstress" may occur and cause cracking in brittle and strain softening materials. Also, the FEM is helpful in identifying how reinforcement will respond in embankments. Finite element analyses may be useful in areas where new types of reinforcement are being used or reinforcement is being used in ways different from the ways for which experience exists. An important input to the stability analyses for reinforced slopes is the force in the reinforcement. The FEM can provide useful guidance for establishing the force that will be used.

*b. Use of finite element analyses to compute factors of safety.* If desired, factors of safety equivalent to those computed using limit equilibrium analyses can be computed from results of finite element analyses. The procedure for using the FEM to compute factors of safety are as follows:

(1) Perform an analysis using the FEM to determine the stresses for the slope.

(2) Select a trial slip surface.

(3) Subdivide the slip surface into segments.

(4) Compute the normal stresses and shear stresses along an assumed slip surface. This requires interpolation of values of stress from the values calculated at Gauss points in the finite element mesh to obtain values at selected points on the slip surface. If an effective stress analysis is being performed, subtract pore pressures to determine the effective normal stresses on the slip surface. The pore pressures are determined from the same finite element analysis if a coupled analysis was performed to compute stresses and deformations. The pore pressures are determined from a separate steady seepage analysis if an uncoupled analysis was performed to compute stresses and deformations.

(5) Use the normal stress and the shear strength parameters, c and $\phi$, or c' and $\phi'$, to compute the available shear strength at points along the shear surface. Use total normal stresses and total stress shear strength parameters for total stress analysis and effective normal stresses and effective stress shear strength parameters for effective stress analyses.

(6) Compute an overall factor of safety using the following equation:

$$F = \frac{\sum s_i \Delta\ell}{\sum \tau_i \Delta\ell} \tag{C-37}$$

where

$s_i$ = available shear strength computed in step (4)

$\tau_i$ = shear stress computed in step (3)

$\Delta\ell$ = length of each individual segment into which the slip surface has been subdivided

The summations in Equation C-37 are performed over all the segments into which the slip surface has been subdivided.

(a) Studies have shown that factors of safety determined using the procedure described are, for practical purposes, equal to factors of safety determined using accurate limit equilibrium methods.

(b) "Local" (point-by-point) factors of safety can also be calculated using the stresses and shear strength properties at selected points in a slope. Some of the local factors of safety will be lower than the overall minimum factor of safety computed from Equation C-37 or limit equilibrium analyses. Local factors of safety of one or less do not necessarily indicate that a slope is unstable. Stresses will be redistributed from points of local failure to other points where the local factor of safety is greater than 1. As long as the overall factor of safety is greater than 1, the slope will be stable.

*c. Advantages and disadvantages*. Where estimates of movements as well as factor of safety are required to achieve design objectives, the effort required to perform finite element analyses can be justified. However, finite element analyses require considerably more time and effort, beyond that required for limit equilibrium analyses and additional data related to stress-strain behavior of materials. Therefore, the use of finite element analyses is not justified for the sole purpose of calculating factors of safety.